

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 51 (2005)  
**Heft:** 3-4: L'enseignement mathématique

**Artikel:** On the geometry of holonomy systems  
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#### Bibliographie

**DOI:** <https://doi.org/10.5169/seals-3602>

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This conjecture, which is true if  $\dim(M) = 2$  (see [BCO, p. 198]), is actually equivalent to the following two conjectures taken together. It is not true for non-homogeneous submanifolds since the normal holonomy is invariant under conformal diffeomorphisms of the ambient space.

- (a) *Let  $M$  be a homogeneous irreducible and full submanifold of the sphere, different from a curve, which is not an orbit of an  $s$ -representation. Then the normal holonomy group acts irreducibly.*
- (b) *Let  $M$  be a homogeneous and full submanifold of the sphere such that the normal holonomy acts irreducibly and is non-transitive. Then  $M$  is an orbit of an  $s$ -representation.*

Corollary 4.2 might be useful in the proof of part (b).

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(Reçu le 24 février 2005)

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Page vide