$$\det \begin{pmatrix} a_1^{(p)} \\ \vdots \\ a_n^{(p)} \end{pmatrix} \neq 0\,.$$

By the Siegel-Watson theorem, one can choose $a'_1, \ldots, a'_n$ in $\mathbf{Z}^n$ such that $f'(a'_i) = b_i$ for $1 \leq i \leq n$ and a'_i is p-adically arbitrarily close to $a_i^{(p)}$. Then

$$\det \begin{pmatrix} a'_1 \\ \vdots \\ a'_n \end{pmatrix} \neq 0\,,$$

and hence $a'_1, \ldots, a'_n$ are linearly independent in $\mathbf{Z}^n$.

We can now complete the proof. Since $D(f) = D(f')$, the fact established above implies $\beta(f) \leq \beta(f')$. Thus $\beta_{n,s} \leq \beta_{n,s'}$, the form f in $\mathcal{E}_{n,s}(\mathbf{Z})$ being arbitrary. Consequently one gets

$$\beta_{n,s} = \beta_{n,s'}$$

by interchanging s and s'.

Finally observe that for s and s' under consideration,

$$w_{n,s} = w_{n,s'}\,.$$

By Corollary 2.3, the last two equalities imply the equivalence of $T(n,s)$ and $T(n,s')$ for $n \geq 3$. If $n = 2$, then $s = s' = 0$, and there is nothing to prove. □

REFERENCES

[1] BARNES, E. S. The minimum of the product of two values of a quadratic form. I. *Proc. London Math. Soc (3) 1* (1951), 257–283.

[2] —— On indefinite ternary quadratic forms. *Proc. London Math. Soc. (3) 2* (1952), 219–233.

[3] CASSELS, J. W. S. *An Introduction to the Geometry of Numbers.* Springer-Verlag, 1971.

[4] —— *Rational Quadratic Forms.* Academic Press, London, 1978.

[5] DANI, S. G. and G. MARGULIS. Values of quadratic forms at integral points: an elementary approach. *L'Enseignement Math. (2) 36* (1990), 143–174.

[6] MINKOWSKI, H. *Geometrie der Zahlen.* Leipzig and Berlin, 1898.

[7] SERRE, J.-P. *A Course in Arithmetic.* Springer-Verlag, 1993.

[8] WATSON, G. L. One-sided inequalities for integral quadratic forms. *Quart. J. Math. Oxford Ser. (2) 9* (1958), 99–108.

[9] —— Asymmetric inequalities for indefinite quadratic forms. *Proc. London Math. Soc. (3) 18* (1968), 95–113.

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