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Proof. Let K be the disjoint union of the $K(G_i, 1)$ for $i \in I$ and form the open wedge K^+ (i.e. add an arc to an external basepoint for each component) and then construct L, a $K(G, 1)$, by attaching cells to K^+.

Let $\widetilde{L}$ be the universal cover of L and $\widetilde{K}$ the inverse image of K in $\widetilde{L}$ (which comprises a number of disjoint copies of universal covers of the $K(G_i, 1)$'s). Then form $\widehat{L}$ by squeezing each component of $\widetilde{K}$ to a point. Then, since we are squeezing contractible subcomplexes, $\widehat{L}$ is contractible and G acts freely off the 0-skeleton. Further the stabilisers of the vertices are the conjugates of the subgroups G_i, so we have to prove that each finite subgroup F of G has a global fixed point.

To do this we use Theorem 1.1 with $Q = \widehat{L}$. The space $\widehat{L}$ is contractible, so we have hypothesis (a). We have to check (b).

Now if H is a subgroup of G then $\widehat{L}/H$ is formed from a cover of L by squeezing components of the preimage of K. But the cohomology hypotheses lift to any cover (since they are given "for any G-module") so $\widehat{L}/H$ is formed by squeezing a subspace which carries all but finitely many of the cohomology groups and hence by excision it has finite (co)homological dimension. Thus we have hypothesis (b) of Theorem 1.1.

REMARK 4.2. The Global Fixed-Point Theorem is stronger than needed to prove the Bogley-Pride or Serre theorems. In these applications Q was contractible instead of just $\mathbf{Z}/p\mathbf{Z}$-acyclic for certain p and $Q/\langle g \rangle$ was either finite dimensional or had finite homological dimension with all coefficients.

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