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Corollary 8.10 is purely arithmetical in nature. It should be possible to give a direct proof, but this probably requires a more ad hoc approach, making for instance a case distinction between the supersingular and the ordinary primes for E over K.

The next corollary is certainly well-known, but it is amusing to see how it can be proved using Arakelov theory.

COROLLARY 8.11. *Suppose that $N = p$ is a prime number. Extend the p-torsion points of E over the regular minimal model of E over K. Then the p-torsion points restrict injectively to a fiber at a prime of characteristic different from p.*

Proof. By symmetry considerations, it suffices to prove that for any p-torsion point P, the sections P and O do not intersect at a fiber above a prime of characteristic different from p. But if we take $N = p$ in the formula from Corollary 8.10, the right hand side is a rational multiple of $\log p$, hence so is the left hand side. As the local intersections involved in $(P, O)_{\mathrm{fin}}$ are always non-negative, they are in fact zero at primes of characteristic different from p. This proves the corollary. □

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REFERENCES

[1] ARAKELOV, S. Y. An intersection theory for divisors on an arithmetic surface. *Math. USSR Izvestija 8* (1974), 1167–1180.

[2] AUTISSIER, P. Hauteur des correspondances de Hecke. *Bull. Soc. Math. France 131* (2003), 421–433.

[3] CASSOU-NOGUÈS, PH. and M. J. TAYLOR. *Elliptic Functions and Rings of Integers. Progr. Math. 66.* Birkhäuser Verlag, 1987.

[4] DELIGNE, P. and D. MUMFORD. The irreducibility of the space of curves of given genus. *Inst. Hautes Études Sci. Publ. Math. 36* (1969), 75–109.

[5] DELIGNE, P. et M. RAPOPORT. Les schémas de modules de courbes elliptiques. In: *Modular Functions of One Variable, II.* Lectures Notes in Mathematics *349.* Springer Verlag, 1973.

[6] FALTINGS, G. Endlichkeitssätze für abelsche Varietäten über Zahlkörpern. *Invent. Math. 73* (1983), 349–366.

[7] —— Calculus on arithmetic surfaces. *Ann. of Math. (2) 119* (1984), 387–424.

[8] LIU, Q. *Algebraic Geometry and Arithmetic Curves*. Oxford Graduate Texts in Mathematics, *6*. Oxford Science Publications, 2002.

[9] MUMFORD, D. *Tata Lectures on Theta I, II. Progr. in Math. 28, 43*. Birkhäuser Verlag, 1984.

[10] RAYNAUD, M. Hauteurs et isogénies. Astérisque *127* (1985), 199–234.

[11] SILVERMAN, J. Heights and elliptic curves. In: *Arithmetic Geometry*. G. Cornell and J. Silverman (eds.). Springer Verlag, 1986.

[12] SZPIRO, L. Sur les propriétés numériques du dualisant relatif d'une surface arithmétique. In: *The Grothendieck Festschrift, Vol. III, 229–246. Progr. Math. 88*. Birkhäuser Verlag, 1990.

[13] SZPIRO, L. and E. ULLMO. Variation de la hauteur de Faltings dans une classe de $\overline{\mathbf{Q}}$-isogénie de courbe elliptique. *Duke Math. J. 97* (1999), 81–97.

[14] TATE, J. Algorithm for determining the type of a singular fiber in an elliptic pencil. In: *Modular Functions of One Variable, IV*. Lecture Notes in Mathematics *476*. Springer Verlag, 1975.

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