

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 51 (2005)
Heft: 3-4: L'enseignement mathématique

Artikel: On the Arakelov theory of elliptic curves
Autor: Jong, Robin de

Bibliographie

DOI: <https://doi.org/10.5169/seals-3593>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 17.05.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Corollary 8.10 is purely arithmetical in nature. It should be possible to give a direct proof, but this probably requires a more ad hoc approach, making for instance a case distinction between the supersingular and the ordinary primes for E over K .

The next corollary is certainly well-known, but it is amusing to see how it can be proved using Arakelov theory.

COROLLARY 8.11. *Suppose that $N = p$ is a prime number. Extend the p -torsion points of E over the regular minimal model of E over K . Then the p -torsion points restrict injectively to a fiber at a prime of characteristic different from p .*

Proof. By symmetry considerations, it suffices to prove that for any p -torsion point P , the sections P and O do not intersect at a fiber above a prime of characteristic different from p . But if we take $N = p$ in the formula from Corollary 8.10, the right hand side is a rational multiple of $\log p$, hence so is the left hand side. As the local intersections involved in $(P, O)_{\text{fin}}$ are always non-negative, they are in fact zero at primes of characteristic different from p . This proves the corollary. \square

ACKNOWLEDGEMENTS. The author wishes to thank Gerard van der Geer for his encouragement and helpful remarks. Also he thanks Professor Qing Liu and the referee for their comments on an earlier version of this paper.

REFERENCES

- [1] ARAKELOV, S. Y. An intersection theory for divisors on an arithmetic surface. *Math. USSR Izvestija* 8 (1974), 1167–1180.
- [2] AUTISSIER, P. Hauteur des correspondances de Hecke. *Bull. Soc. Math. France* 131 (2003), 421–433.
- [3] CASSOU-NOGUÈS, PH. and M.J. TAYLOR. *Elliptic Functions and Rings of Integers*. Progr. Math. 66. Birkhäuser Verlag, 1987.
- [4] DELIGNE, P. and D. MUMFORD. The irreducibility of the space of curves of given genus. *Inst. Hautes Études Sci. Publ. Math.* 36 (1969), 75–109.
- [5] DELIGNE, P. et M. RAPOPORT. Les schémas de modules de courbes elliptiques. In: *Modular Functions of One Variable, II*. Lectures Notes in Mathematics 349. Springer Verlag, 1973.
- [6] FALTINGS, G. Endlichkeitssätze für abelsche Varietäten über Zahlkörpern. *Invent. Math.* 73 (1983), 349–366.
- [7] —— Calculus on arithmetic surfaces. *Ann. of Math.* (2) 119 (1984), 387–424.

- [8] LIU, Q. *Algebraic Geometry and Arithmetic Curves*. Oxford Graduate Texts in Mathematics, 6. Oxford Science Publications, 2002.
- [9] MUMFORD, D. *Tata Lectures on Theta I, II*. Progr. in Math. 28, 43. Birkhäuser Verlag, 1984.
- [10] RAYNAUD, M. Hauteurs et isogénies. Astérisque 127 (1985), 199–234.
- [11] SILVERMAN, J. Heights and elliptic curves. In: *Arithmetic Geometry*. G. Cornell and J. Silverman (eds.). Springer Verlag, 1986.
- [12] SZPIRO, L. Sur les propriétés numériques du dualisant relatif d'une surface arithmétique. In: *The Grothendieck Festschrift, Vol. III*, 229–246. Progr. Math. 88. Birkhäuser Verlag, 1990.
- [13] SZPIRO, L. and E. ULLMO. Variation de la hauteur de Faltings dans une classe de $\overline{\mathbb{Q}}$ -isogénie de courbe elliptique. *Duke Math. J.* 97 (1999), 81–97.
- [14] TATE, J. Algorithm for determining the type of a singular fiber in an elliptic pencil. In: *Modular Functions of One Variable, IV*. Lecture Notes in Mathematics 476. Springer Verlag, 1975.

(Reçu le 6 septembre 2004)

R. de Jong

Mathematical Institute
 University of Leiden
 PO Box 9512
 2300 RA Leiden
 The Netherlands
e-mail: rdejong@math.leidenuniv.nl

Leere Seite
Blank page
Page vide