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det = 18: Now assume that $\det \tilde{M} = 18$. Then \tilde{M} is a maximal integral lattice (since there is no even lattice of determinant 2 in dimension 13). In particular the discriminant group of \tilde{M} is not cyclic, and the 3-Sylow subgroup of \tilde{M}^*/\tilde{M} is isometric to the unique anisotropic quadratic space of dimension 2 over \mathbb{F}_3 . There is a unique genus of such lattices, namely that of $E_6 \perp E_6 \perp A_1$. This genus has class number 7. Only the lattice $E_6 \perp E_6 \perp A_1$ satisfies condition (MIN). Only one sublattice of index 9 of this lattice satisfies condition (MIN) and this lattice is isometric to $A_1 \perp CT$.

det = 32: If $\det \tilde{M} = 32$, then \tilde{M} has an even overlattice $\tilde{\tilde{M}}$ of determinant 8. By [6, Table 15.4] there is a unique genus of even lattices of determinant 8 in dimension 13, the one of $A_1 \perp D_4 \perp E_8$. This genus contains 4 classes, none of which satisfies condition (MIN).

det = 48: Assume finally that $\det(\tilde{\tilde{M}}) = 48$. Then there is an even overlattice $\tilde{\tilde{\tilde{M}}}$ of $\tilde{\tilde{M}}$ of determinant $\det(\tilde{\tilde{\tilde{M}}}) = 12$. There is one genus of such lattices of determinant 12, namely the one of $E_6 \perp D_7$. Its class number is 7 and none of the lattices satisfies condition (MIN). \square

Together with Proposition 3.36 this concludes the proof of Theorem 3.22.

REFERENCES

- [1] ANSTREICHER, K.M. Improved linear programming bounds for antipodal spherical codes. *Discrete Comput. Geom.* 28 (2002), 107–114.
- [2] BACHOC, C. and B. VENKOV. Modular forms, lattices and spherical designs. *Monogr. Enseign. Math.* 37 (2001), 87–111.
- [3] BANNAI, E., A. MUNEMASA and B. VENKOV. The nonexistence of certain tight spherical designs. *Algebra i Analiz* 16 (2004), 1–23.
- [4] COHN, H. and N.D. ELKIES. New upper bounds on sphere packings I. *Ann. of Math. (2)* 157 (2003), 689–714 (math.MG/0110009).
- [5] COHN, H. and A. KUMAR. Optimality and uniqueness of the Leech lattice among lattices. Preprint, 2003 (math.MG/0403263).
- [6] CONWAY, J.H. and N.J.A. SLOANE. *Sphere Packings, Lattices and Groups*, 3rd ed. Springer-Verlag, 1998.
- [7] CONWAY, J.H. and N.J.A. SLOANE. Low dimensional lattices VI: Voronoi reduction of three-dimensional lattices. *Proc. Roy. Soc. London, Ser. A*, 436 (1992), 55–68.
- [8] DE LA HARPE, P. and C. PACHE. Cubature formulas, geometrical designs, reproducing kernels and Markov operators. Preprint, Université Genève, 2004 (<http://www.unige.ch/math/biblio/?preprint/liste.html>).
- [9] KING, O. A mass formula for unimodular lattices with no roots. *Math. Comp.* 72 (2003), 839–863.

- [10] LEMPKEN, W., B. SCHRÖDER and P.-H. TIEP. Symmetric squares, spherical designs, and lattice minima. *J. Algebra* 240 (2001), 185–208.
- [11] MAGMA. *The Magma Computational Algebra System for Algebra, Number Theory and Geometry*, available via the magma home page <http://www.maths.usyd.edu.au:8000/u/magma/>
- [12] MARTINET, J. *Les réseaux parfaits des espaces euclidiens*. Masson, 1996. English edition: *Perfect Lattices in Euclidean Spaces*. Grundlehren der math. Wissenschaften 327. Springer-Verlag, Berlin, 2003.
- [13] NEBE, G. and B. VENKOV. The strongly perfect lattices of dimension 10. *J. Théor. Nombres Bordeaux* 12 (2000), 503–518.
- [14] QUEBBEMANN, H.-G. Modular lattices in euclidean spaces. *J. Number Theory* 54 (1995), 190–202.
- [15] SHEPHARD, G. C. and J. A. TODD. Finite unitary reflection groups. *Canadian J. Math.* 6 (1954), 274–304.
- [16] VENKOV, B. Even unimodular extremal lattices. *Proc. Steklov Inst.* 165 (1984), 47–52.
- [17] ——— Réseaux et designs sphériques. *Monogr. Enseign. Math.* 37 (2001), 10–86.
- [18] VORONOI, G. Nouvelles applications des paramètres continus à la théorie des formes quadratiques: 1. Sur quelques propriétés des formes quadratiques positives parfaites. *J. reine angew. Math.* 133 (1908), 97–178.

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