det = 18 : Now assume that $\det \widetilde{M} = 18$. Then $\widetilde{M}$ is a maximal integral lattice (since there is no even lattice of determinant 2 in dimension 13). In particular the discriminant group of $\widetilde{M}$ is not cyclic, and the 3-Sylow subgroup of $\widetilde{M}^* / \widetilde{M}$ is isometric to the unique anisotropic quadratic space of dimension 2 over $\mathbf{F}_3$. There is a unique genus of such lattices, namely that of $E_6 \perp E_6 \perp A_1$. This genus has class number 7. Only the lattice $E_6 \perp E_6 \perp A_1$ satisfies condition (MIN). Only one sublattice of index 9 of this lattice satisfies condition (MIN) and this lattice is isometric to $A_1 \perp CT$.

det = 32 : If $\det \widetilde{M} = 32$, then $\widetilde{M}$ has an even overlattice $\widetilde{\widetilde{M}}$ of determinant 8. By [6, Table 15.4] there is a unique genus of even lattices of determinant 8 in dimension 13, the one of $A_1 \perp D_4 \perp E_8$. This genus contains 4 classes, none of which satisfies condition (MIN).

det = 48 : Assume finally that $\det(\widetilde{M}) = 48$. Then there is an even overlattice $\widetilde{\widetilde{M}}$ of $\widetilde{M}$ of determinant $\det(\widetilde{\widetilde{M}}) = 12$. There is one genus of such lattices of determinant 12, namely the one of $E_6 \perp D_7$. Its class number is 7 and none of the lattices satisfies condition (MIN). □

Together with Proposition 3.36 this concludes the proof of Theorem 3.22.

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