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(2.9) $$c' \left| (f(\phi_t(x)) - b)^{1-\alpha} - (f(\phi_t(y)) - b)^{1-\alpha} \right| < \varepsilon/4 .$$

This can be done because both maps $z \mapsto \phi_t(z)$ and $z \mapsto (f(\phi_t(z)) - b)^{1-\alpha}$ are continuous. Equations (2.6) and (2.9) imply that

(2.10) $$c'(f(\phi_t(y)) - b)^{1-\alpha} < \varepsilon/2 ,$$

and so, by (2.5),

(2.11) $$d(\phi_\infty(y), \phi_t(y)) < \varepsilon/2 .$$

Putting (2.7), (2.8) and (2.10) together, we get that for $y \in S_C$, $d(x,y) < \delta$ implies

$$\begin{aligned} d(\phi_\infty(x), \phi_\infty(y)) &\leq d(\phi_\infty(x), \phi_t(x)) + d(\phi_t(x), \phi_t(y)) + d(\phi_t(y), \phi_\infty(y)) \\ &< \varepsilon/4 + \varepsilon/4 + \varepsilon/2 = \varepsilon. \end{aligned}$$

This proves that $\phi_\infty \colon S_C \to C$ is continuous.

Finally it follows from the argument above that for any $y_0 \in S_C$ and any $\varepsilon > 0$ there are $\delta > 0$ and $\tau > 0$ so that

$$t > \tau \text{ and } d(y, y_0) < \delta \Rightarrow d(\phi_t(y), \phi_\infty(y_0)) < \varepsilon$$

for all $y \in S_C$. Consequently

$$\phi \colon [0, \infty] \times S_C \to S_C, \quad (t, y) \mapsto \phi_t(y)$$

is continuous. That is, S_C deformation retracts onto C. This concludes the proof of Theorem 1.1. □

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