

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 51 (2005)
Heft: 1-2: L'enseignement mathématique

Artikel: Gradient flow of the norm squared of a moment map
Autor: Lerman, Eugene

Bibliographie
DOI: <https://doi.org/10.5169/seals-3590>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 27.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

$$(2.9) \quad c' |(f(\phi_t(x)) - b)^{1-\alpha} - (f(\phi_t(y)) - b)^{1-\alpha}| < \varepsilon/4.$$

This can be done because both maps $z \mapsto \phi_t(z)$ and $z \mapsto (f(\phi_t(z)) - b)^{1-\alpha}$ are continuous. Equations (2.6) and (2.9) imply that

$$(2.10) \quad c'(f(\phi_t(y)) - b)^{1-\alpha} < \varepsilon/2,$$

and so, by (2.5),

$$(2.11) \quad d(\phi_\infty(y), \phi_t(y)) < \varepsilon/2.$$

Putting (2.7), (2.8) and (2.10) together, we get that for $y \in S_C$, $d(x, y) < \delta$ implies

$$\begin{aligned} d(\phi_\infty(x), \phi_\infty(y)) &\leq d(\phi_\infty(x), \phi_t(x)) + d(\phi_t(x), \phi_t(y)) + d(\phi_t(y), \phi_\infty(y)) \\ &< \varepsilon/4 + \varepsilon/4 + \varepsilon/2 = \varepsilon. \end{aligned}$$

This proves that $\phi_\infty: S_C \rightarrow C$ is continuous.

Finally it follows from the argument above that for any $y_0 \in S_C$ and any $\varepsilon > 0$ there are $\delta > 0$ and $\tau > 0$ so that

$$t > \tau \text{ and } d(y, y_0) < \delta \Rightarrow d(\phi_t(y), \phi_\infty(y_0)) < \varepsilon$$

for all $y \in S_C$. Consequently

$$\phi: [0, \infty] \times S_C \rightarrow S_C, \quad (t, y) \mapsto \phi_t(y)$$

is continuous. That is, S_C deformation retracts onto C . This concludes the proof of Theorem 1.1. \square

REFERENCES

- [AB] ATIYAH, M. and R. BOTT. The Yang-Mills equations over Riemann surfaces. *Philos. Trans. Roy. Soc. Lond. Ser. A*, 308 (1983), 523–615.
- [BM] BIERSTONE, E. and P. MILMAN. Semianalytic and subanalytic sets. *Inst. Hautes Études Sci. Publ. Math.* 67 (1988), 5–42.
- [Bo] BOTT, R. Morse theory indomitable. *Inst. Hautes Études Sci. Publ. Math.* 68 (1988), 99–114.

- [Da] DASKALOPOULOS, G. The topology of the space of stable bundles on a compact Riemann surface. *J. Differential Geom.* 36 (1992), no. 3, 699–746.
- [Do] DONALDSON, S. K. A new proof of a theorem of Narasimhan and Seshadri. *J. Differential Geom.* 18 (1983), no. 2, 269–277.
- [GS1] GUILLEMIN, V. and S. STERNBERG. Geometric quantization and multiplicities of group representations. *Invent. Math.* 67 (1982), 515–538.
- [GS2] GUILLEMIN, V. and S. STERNBERG. A normal form for the moment map. In: *Differential geometric methods in mathematical physics* (Jerusalem, 1982). *Mathematical Physics Studies* 6, 161–175. Reidel, Dordrecht, 1984.
- [KN] KEMPF, G. and L. NESS. The length of vectors in representation spaces. In: *Algebraic Geometry. Proc. Summer Meeting, Univ. Copenhagen, Copenhagen* (1978). *Lecture Notes in Math.* 732, 233–243. Springer, Berlin, 1979.
- [K] KIRWAN, F. *Cohomology of Quotients in Symplectic and Algebraic Geometry*. Princeton Univ. Press, Princeton (N.J.), 1984.
- [KMP] KURDYKA, K., T. MOSTOWSKI and A. PARUSIŃSKI. Proof of the gradient conjecture of R. Thom. *Ann. of Math. (2)* 152 (2000), 763–792, arXiv: math.AG/9906212.
- [Lo] ŁOJASIEWICZ, S. Ensembles semi-analytiques. *IHES preprint*, 1965.
- [Lo2] — Sur les trajectoires du gradient d’une fonction analytique. *Seminari di Geometria, Bologna* (1982/83), Università degli Studi di Bologna, Bologna, 1984, 115–117.
- [Ma] MARLE, C.-M. Le voisinage d’une orbite d’une action hamiltonienne d’un groupe de Lie. In: *Séminaire sud-rhodanien de géométrie, II* (Lyon, 1983), 19–35, Travaux en cours. Hermann, Paris, 1984.
- [MW] MARSDEN, J. and A. WEINSTEIN. Reduction of symplectic manifolds with symmetry. *Rep. Mathematical Phys.* 5 (1974), 121–130.
- [Me] MEYER, K. Symmetries and integrals in mechanics. In: *Dynamical systems. Proc. Sympos., Univ. Bahia, Salvador, 1971*, 259–272. Academic Press, New York, 1973.
- [MFK] MUMFORD, D., J. FOGARTY and F. KIRWAN. *Geometric Invariant Theory*. Third edition. *Ergebnisse der Mathematik und ihrer Grenzgebiete* (2) 34. Springer-Verlag, Berlin, 1994.
- [Ne] NEEMAN, A. The topology of quotient varieties. *Ann. of Math. (2)*, 122 (1985), no. 3, 419–459.
- [N] NESS, L. A stratification of the null cone via the moment map. *Amer. J. Math.* 106 (1984), no. 6, 1281–1329.
- [PdM] PALIS, J. JR. and W. DE MELO. *Geometric Theory of Dynamical Systems. An Introduction*. Translated from the Portuguese by A. K. Manning. Springer-Verlag, New York-Berlin, 1982.
- [S] SCHWARZ, G. The topology of algebraic quotients. In: *Topological Methods in Algebraic Transformation Groups* (New Brunswick, NJ, 1988), 135–151. *Progr. Math.* 80, Birkhäuser Boston, Boston, MA, 1989.
- [SL] SJAMAAR, R. and E. LERMAN. Stratified symplectic spaces and reduction. *Ann. of Math. (2)* 134 (1991), 375–422.

- [W] WOODWARD, C. The Yang-Mills heat flow on the moduli space of framed bundles on a surface, math.SG/0211231.

(Reçu le 27 octobre 2004)

Eugene Lerman

University of Illinois

Urbana, IL 61801

U. S. A.

e-mail: lerman@math.uiuc.edu

Australian National University, Canberra, ACT 0200

Leere Seite

Blank page

Page vide