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GRADIENT FLOW OF THE NORM SQUARED OF A MOMENT MAP

by Eugene LERMAN *)

ABSTRACT. We present a proof due to Duistermaat that the gradient flow of the norm squared of the moment map defines a deformation retract of the appropriate piece of the manifold onto the zero level set of the moment map. Duistermaat's proof is an adaptation of Łojasiewicz's argument for analytic functions to functions which are locally analytic.

1. INTRODUCTION

Recall that a 2-form σ on a manifold M is *symplectic* if it is closed and nondegenerate. Recall also that if a Lie group G acts on a manifold M then for any element X in the Lie algebra $\mathfrak{g}$ of G we get a vector field X_M on M defined by:

$$X_M(m) = \left.\frac{d}{dt}\right|_{t=0} \exp tX \cdot m$$

for all $m \in M$. Here $\exp\colon \mathfrak{g} \to G$ denotes the exponential map and "$\cdot$" the action. Given an action of a Lie group G on a manifold M preserving a symplectic form σ there may exist a *moment map* $\mu\colon M \to \mathfrak{g}^*$ associated with this action. It takes its values in $\mathfrak{g}^* := \mathrm{Hom}(\mathfrak{g}, \mathbf{R})$, in the dual of the Lie algebra of G. The moment map is defined as follows: for any $X \in \mathfrak{g}$, $\langle \mu, X\rangle$ is a real-valued function on M ($\langle \cdot, \cdot \rangle$ denotes the canonical pairing $\mathfrak{g}^* \times \mathfrak{g} \to \mathbf{R}$). We require that

$$d\langle \mu, X\rangle = \sigma(X_M, \cdot)$$

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for any [1]) $X \in \mathfrak{g}$. Additionally one often requires that the moment map μ is *equivariant*, that is, that it intertwines the given action of G on M with the coadjoint action of G on $\mathfrak{g}^*$:

$$\mu(g \cdot m) = g \cdot \mu(m)$$

for all $g \in G$ and $m \in M$. An equivariant moment map is unique up to an addition of a vector in $(\mathfrak{g}^*)^G$, the G-fixed subspace of $\mathfrak{g}^*$. For example if G is a subgroup of the unitary group $U(n)$, then its natural action on the projective space $\mathbf{C}P^{n-1}$ not only preserves the imaginary part σ of the Fubini-Study metric but is, in fact, Hamiltonian. The associated moment map μ is given by

$$\langle \mu([v]), X \rangle = \frac{1}{\|v\|^2}(Xv, v),$$

where $[v] \in \mathbf{C}P^{n-1}$ denotes the line through a vector $v \in \mathbf{C}^n \setminus 0$, $(\cdot,\cdot)$ denotes the standard Hermitian inner product on $\mathbf{C}^n$ and Xv denotes the image of v under $X \in \mathfrak{g} \subset \operatorname{Hom}(\mathbf{C}^n, \mathbf{C}^n)$ (see for instance [K, p. 24]). If $V \subset \mathbf{C}P^{n-1}$ is a G-invariant smooth subvariety, then the action of G on V also has a moment map and it is simply the restriction $\mu|_V$.

Equivariant moment maps can be used to construct new symplectic manifolds: Suppose that the Lie group G acts freely and properly on the zero level set $\mu^{-1}(0)$ of an associated moment map $\mu \colon M \to \mathfrak{g}^*$. Then by a theorem of Meyer [Me] and, independently, of Marsden and Weinstein [MW],

$$M/\!/_0 G := \mu^{-1}(0)/G$$

is a symplectic manifold, called the *symplectic quotient* at 0. If the action of G on $\mu^{-1}(0)$ is not free then $M/\!/_0 G$ is a stratified space with symplectic strata [SL].

In many cases one knows the topology of M and one is interested in understanding the topology of the symplectic quotient $M/\!/_0 G$, which may be much more complicated. For instance a projective toric manifold is a symplectic quotient of a symplectic vector space by a theorem of Delzant. Such a manifold has an interesting cohomology ring, while the vector space one constructs it from is contractible.

A more interesting example is due to Atiyah and Bott [AB]. Consider the space $\mathcal{A}$ of connections on a vector bundle E over a Riemann surface Σ. Then $\mathcal{A}$ is an affine infinite dimensional space with a constant coefficients symplectic form. Moreover the action of the gauge group $\mathcal{G}$ on $\mathcal{A}$ is Hamiltonian and

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[1]) The opposite sign convention is also used by many authors: $d\langle \mu, X \rangle = -\sigma(X_M, \cdot)$.

the associated moment map μ assigns to a connection A its curvature F_A. Therefore the symplectic quotient $\mathcal{A}//_0\mathcal{G}$ is (formally) the moduli space of flat connections. Atiyah and Bott also pointed out that the norm squared $||\mu||^2$ of the moment map behaves somewhat like a very nice Morse-Bott function. While the connected components of the critical set of $||\mu||^2$ are not manifolds, they do have indices. Moreover, for each critical component C the set S_C given by

$$S_C := \{x \in M \mid \omega\text{-limit of the trajectory of } -\nabla||\mu||^2 \text{ is in } C\} \tag{1.1}$$

is a manifold — the "stable manifold" of C. The manifolds S_C give rise to a decomposition of $\mathcal{A}$. This was later made rigorous by Donaldson [Do] and Daskalopoulos [Da]. In the finite dimensional setting these ideas were developed by Kirwan [K] and Ness [N] (independently of each other).

The work of Kirwan and Ness had an additional motivation that comes from the Geometric Invariant Theory (GIT) of Mumford. Roughly speaking given a complex projective variety $M \subset \mathbf{P}(V)$ and a complex reductive group $G^{\mathbf{C}} \subset \mathrm{PGL}(V)$ one can form a new projective variety $M//G^{\mathbf{C}}$, the GIT quotient of M. It is obtained by taking a Zariski dense subset $M_{ss} \subset M$ of "semi-stable points" and dividing out by $G^{\mathbf{C}}$:

$$M//G^{\mathbf{C}} := M_{ss}/G^{\mathbf{C}}.$$

It turns out that the action of the maximal compact subgroup G of $G^{\mathbf{C}}$ on M is Hamiltonian and that the two quotients are equal:

$$M//G^{\mathbf{C}} = M//_0G. \tag{1.2}$$

As far as credit for this observation goes, let me quote from a wonderful paper by R. Bott [Bo, p. 112]

> In fact, it is quite distressing to see how long it is taking us collectively to truly sort out symplectic geometry. I became aware of this especially when one fine afternoon in 1980, Michael Atiyah and I were trying to work in my office at Harvard. I say trying, because the noise in the neighboring office made by Sternberg and Guillemin made it difficult. So we went next door to arrange a truce and in the process discovered that we were *grosso modo* doing the same thing. Later Mumford joined us, and before the afternoon was over we saw how Mumford's "stability theory" fitted with Morse theory.

Both Guillemin and Sternberg in [GS1] and Ness in [N] credit Mumford for (1.2).

As I mentioned earlier, it is of some interest in symplectic and algebraic geometry to understand the topology of symplectic and GIT quotients. In particular it is natural to ask whether the stable manifolds S_C retract onto the corresponding critical sets C under the flow of $-\nabla\|\mu\|^2$. This is not entirely obvious in light of the fact that there are functions f on $\mathbf{R}^2$ whose gradient flows have nontrivial ω-limit sets. As the referee pointed out, an example can be found on p. 14 of [PdM]: The function f in question is given in polar coordinates by

$$f(r,\theta) = \begin{cases} e^{\frac{1}{r^2-1}}, & \text{if } r < 1\,; \\ 0, & \text{if } r = 1\,; \\ e^{-\frac{1}{r^2-1}} \sin(1/(r-1)-\theta), & \text{if } r > 0\,. \end{cases}$$

The whole unit circle $\{r = 1\}$ is the ω-limit set of a gradient trajectory of f. For this function the gradient flow does not give rise to a map from S_C to C, let alone a retraction.

If the function f is analytic then the ω-limit sets of the flow ∇f are single points, as was proved by Łojasiewicz [Lo2]. Pushing this idea a bit further and using the results of Kempf and Ness [KN] Neeman proved that in algebraic setting the flow of $-\nabla\|\mu\|^2$ defines a retraction of the set of semi-stable points M_{ss} onto the zero set of the moment map $\mu^{-1}(0)$ [Ne]. See also Schwarz [S] for a nice survey. Note that the moment map itself does not appear explicitly in these two papers. The connection is explained in a paper of Linda Ness [N] quoted earlier. In the setting of connections of vector bundles over Riemann surfaces Daskalopoulos [Da] showed that the gradient flow of the $\|\mu\|^2$ defines a continuous deformation retract of the Atiyah-Bott strata S_C onto the components of the critical set of $\|\mu\|^2$.

This leaves us with the question: is it true that for an arbitrary moment map μ the stable manifolds S_C defined by (1.1) retract onto the critical sets C under the flow of $-\nabla\|\mu\|^2$? The answer, under reasonable assumptions on μ, is yes. It has been known to experts for some time. It was proved by Duistermaat in the 1980's. The existence of Duistermaat's proof is even footnoted in [MFK] on p. 166 (this was kindly pointed out by the referee). The result has been used by a number of authors, but until Woodward wrote it up in [W, Appendix B], there was no widely available proof. In this paper I do my best to write out Duistermaat's proof in detail. I do not claim any originality. The proof closely follows ideas of Łojasiewicz [Lo2] on the properties of gradient flows of analytic functions (see [KMP], p. 763 and p. 765 for a nice summary). The norm squared of a moment map is not necessarily an analytic function, but it is one locally. The latter is enough to prove that the Łojasiewicz gradient inequality (see Lemma 2.2 below) holds for the norm squared of the moment map and make the rest Łojasiewicz's argument work.

Woodward proves the Łojasiewicz inequality directly using the local normal form theorem for moment maps. He gets more precise constants for the rate of convergence of the gradient flow but his exposition is a bit terse.

We now recapitulate our notation and make things a bit more precise. Let (M,σ) be a connected symplectic manifold with a Hamiltonian action of a compact Lie group K and an associated equivariant moment map $\mu\colon M \to \mathfrak{k}^*$. We assume throughout the rest of paper that the moment map is *proper*. We fix an invariant inner product on $\mathfrak{k}^*$ and a Riemannian metric on M compatible with σ. Let

$$f(x) = ||\mu(x)||^2$$

denote the norm squared of the moment map. It is a proper map. We denote the flow of $-\nabla f$ by ϕ_t. Thus, for any function h and any $x \in M$ we have

$$\frac{d}{dt} h(\phi_t(x)) = -\nabla f(h)\,(\phi_t(x)) = -(\nabla f \cdot \nabla h)\,(\phi_t(x))\,.$$

Since f is proper, the flow exists of all $t \geq 0$. Kirwan proved [K, Theorem 4.16, p. 56] that the set of the critical points of the function f is a disjoint union of path connected closed subsets on each of which f takes constant value. Moreover for each such component C the corresponding stable set S_C defined by

$$S_C := \{x \in M \mid \omega\text{-limit } \phi_t(x) \subset C\}$$

is a smooth manifold. The main result of the paper is

THEOREM 1.1 (Duistermaat). *Let (M,σ), f, C, ϕ_t and S_C be as above. Then*

1. *for each $x \in M$ the ω-limit set of the trajectory $\phi_t(x)$ is a single point, which we denote by $\phi_\infty(x)$;*
2. *for each connected component C of critical points of f the map*

$$\phi\colon [0,\infty] \times S_C \to C, \quad (t,x) \mapsto \phi_t(x)$$

is a deformation retraction.

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2. Proof of Duistermaat's theorem

We prove Theorem 1.1 in a number of steps. We first argue that moment maps for compact Lie groups are locally real analytic. This local analyticity is a consequence of the local normal form theorem of Marle [Ma] and of Guillemin-Sternberg [GS2] for moment maps and of the fact that compact Lie groups are real analytic. To state the local normal form theorem we need to set up some notation. Let $x \in M$ be a point, G_x be its isotropy group with Lie algebra $\mathfrak{g}_x$, G_α the isotropy group of $\alpha = \mu(x)$ with Lie algebra $\mathfrak{g}_\alpha$. Since the action of G is proper, the isotropy group G_x is compact. We can then choose a G_x-equivariant splitting

$$\mathfrak{g}^* = \mathfrak{g}_x^* \times (\mathfrak{g}_\alpha/\mathfrak{g}_x)^* \times (\mathfrak{g}/\mathfrak{g}_\alpha)^*$$

and thereby the embeddings $\mathfrak{g}_x^* \hookrightarrow \mathfrak{g}^*$ and $(\mathfrak{g}_\alpha/\mathfrak{g}_x)^* \hookrightarrow \mathfrak{g}^*$.

Theorem 2.1 (Marle; Guillemin and Sternberg). *There is a finite-dimensional symplectic representation V of G_x, the associated quadratic homogeneous moment map $\mu_V\colon V \to \mathfrak{g}_x^*$, a neighborhood U of the orbit $G \cdot x \subset M$, a neighborhood U_0 of the zero section of the vector bundle $G \times_{G_x} \big((\mathfrak{g}_\alpha/\mathfrak{g}_x)^* \times V\big) \to G/G_x$ and a diffeomorphism $\phi\colon U_0 \to U$ so that*

$$\mu \circ \phi([g, \eta, v]) = Ad^\dagger(g)(\alpha + \eta + \mu_V(v))$$

for all $[g, \eta, v] \in U_0$. Here $Ad^\dagger$ denotes the co-adjoint action and $[g, \eta, v]$ denotes the orbit of $(g, \eta, v) \in G \times \big((\mathfrak{g}_\alpha/\mathfrak{g}_x)^* \times V\big)$ in the associated bundle $G \times_{G_x} \big((\mathfrak{g}_\alpha/\mathfrak{g}_x)^* \times V\big)$.

Now, since G is a compact Lie group, it is real analytic. A choice of a local analytic section of $G \to G/G_x$ gives coordinates on $G \times_{G_x} \big((\mathfrak{g}_\alpha/\mathfrak{g}_x)^* \times V\big)$ making $\mu \circ \phi$ into a real analytic map.

Next we recall the gradient inequality of Łojasiewicz.

Lemma 2.2 (Łojasiewicz gradient inequality). *If f be a real analytic function on an open set $W \subset R^n$ then for every critical point x of f there are a neighborhood U_x of x and constants $c_x > 0$ and α_x, $0 < \alpha_x < 1$, such that*

$$\|\nabla f(y)\| \geq c_x |f(y) - f(x)|^{\alpha_x} \tag{2.1}$$

for all $y \in U_x$. Here $\|\cdot\|$ denotes the standard Euclidean norm.

Proof. This is Proposition 1 on p. 92 of [Lo]. Alternatively see Proposition 6.8 in [BM].

Since any Riemannian metric on a relatively compact subset of $\mathbf{R}^n$ is equivalent to the Euclidean metric, the inequality (2.1) holds for an arbitrary Riemannian metric on $\mathbf{R}^n$ with the *same* exponent α_x and possibly different constant c_x.

Since the connected component C of the set of critical points of f is compact, it can be covered by finitely many open sets U_i on which the inequality (2.1) holds (with the constants c_i and α_i depending on the U_i). Let $\alpha = \max \alpha_i$ and $b = f(C)$. Then for any $y \in U_i$ with $|f(y) - b| < 1$ we have

$$|f(y) - b|^{\alpha_i} \geq |f(y) - b|^{\alpha} .$$

Let $c = \min c_i$ and let $U = \bigcup U_i \cap \left\{ z \in M \;\middle|\; |f(z) - b| < 1 \right\}$. Then for any $y \in U$ we have $\|\nabla f(y)\| \geq c|f(y) - b)|^{\alpha}$ or, equivalently,

$$\|\nabla f(y)\| \cdot |f(y) - b)|^{-\alpha} \geq c . \tag{2.2}$$

By definition of S_C,

$$\lim_{t \to +\infty} f(\phi_t(y)) = b$$

for any $y \in S_C$. Since f is proper, there is $D > 0$ so that

$$|f(y) - b| < D \Rightarrow y \in U .$$

Hence for any $y \in S_C$ there is $\tau = \tau(y)$ so that

$$t \geq \tau(y) \Rightarrow \phi_t(y) \in U .$$

Fix $y \in S_C$. For $t > \tau(y)$

$$\begin{aligned} -\frac{d}{dt}(f(\phi_t(y)) - b)^{1-\alpha} &= -(1 - \alpha)\,(f(\phi_t(y)) - b)^{-\alpha}\,(-\nabla f(f))(\phi_t(y)) \\ &= (1 - \alpha)\,(f(\phi_t(y)) - b)^{-\alpha}\,\|\nabla f(\phi_t(y))\|^2 \\ &\geq (1 - \alpha)\,c\,\|\nabla f(\phi_t(y))\| , \end{aligned}$$

where the last inequality follows by (2.2). Hence for any $t_1 > t_0 > \tau(y)$

$$\begin{aligned} (f(\phi_{t_0}(y)) - b)^{1-\alpha} - (f(\phi_{t_1}(y)) - b)^{1-\alpha} &= -\int_{t_0}^{t_1} \frac{d}{dt}(f(\phi_t(y)) - b)^{1-\alpha}\,dt \\ &\geq (1 - \alpha)c \int_{t_0}^{t_1} \|\nabla f(\phi_t(y))\|\,dt \end{aligned}$$

by the previous inequality. Setting $c' = \frac{1}{(1-\alpha)c}$ we get:

LEMMA 2.3. *There are constants $c' > 0$ and $0 < \alpha < 1$ such that for any $t_0 < t_1$ sufficiently large and any $x \in S_C$*

$$(2.3) \qquad c'\left(\left(f(\phi_{t_0}(x)) - b\right)^{1-\alpha} - \left(f(\phi_{t_1}(x)) - b\right)^{1-\alpha}\right) \geq \int_{t_0}^{t_1} \|\nabla f(\phi_t(x))\|\, dt\,.$$

It is now easy to show that the ω-limit of $\phi_t(y)$ as $t \to +\infty$ is a single point. Denote by d the distance on M defined by the Riemannian metric. Then

$$(2.4) \qquad \begin{aligned} d(\phi_{t_0}(y), \phi_{t_1}(y)) &\leq \int_{t_0}^{t_1} \left\|\frac{d}{dt}(\phi_t(y))\right\| dt \\ &= \int_{t_0}^{t_1} \|\nabla f(\phi_t(y))\|\, dt \\ &\leq c'\left(\left(f(\phi_{t_0}(y)) - b\right)^{1-\alpha} - \left(f(\phi_{t_1}(y)) - b\right)^{1-\alpha}\right). \end{aligned}$$

As $t_0, t_1 \to +\infty$ the last expression converges to 0. Therefore, by the Cauchy criterion, $\lim_{t\to+\infty} \phi_t(y)$ does exist, i.e., the map

$$\phi_\infty : S_C \to C, \quad \phi_\infty(y) := \lim_{t\to+\infty} \phi_t(y)$$

is a well-defined map.

We argue that $\phi_\infty : S_C \to C$ is continuous. Given $x \in S_C$ and $\varepsilon > 0$ we want to find δ such that

$$d(x, y) < \delta \Rightarrow d(\phi_\infty(x), \phi_\infty(y)) < \varepsilon$$

for any $y \in S_C$. If we take the limit of both sides of (2.4) as $t_1 \to +\infty$ we get

$$(2.5) \qquad d(\phi_t(y), \phi_\infty(y)) \leq c'\left(f(\phi_t(y)) - b\right)^{1-\alpha}$$

for all $y \in S_C$ and all t sufficiently large. Choose $t > 0$ so that

$$(2.6) \qquad c'(f(\phi_t(x)) - b)^{1-\alpha} < \varepsilon/4\,.$$

Then

$$(2.7) \qquad d(\phi_t(x), \phi_\infty(x)) < \varepsilon/4\,.$$

With t fixed as above, choose $\delta > 0$ so that $y \in S_C$ and $d(x, y) < \delta$ imply two inequalities:

$$(2.8) \qquad d(\phi_t(x), \phi_t(y)) < \varepsilon/4\,,$$

and

$$c'\left|(f(\phi_t(x)) - b)^{1-\alpha} - (f(\phi_t(y)) - b)^{1-\alpha}\right| < \varepsilon/4. \tag{2.9}$$

This can be done because both maps $z \mapsto \phi_t(z)$ and $z \mapsto (f(\phi_t(z)) - b)^{1-\alpha}$ are continuous. Equations (2.6) and (2.9) imply that

$$c'(f(\phi_t(y)) - b)^{1-\alpha} < \varepsilon/2, \tag{2.10}$$

and so, by (2.5),

$$d(\phi_\infty(y), \phi_t(y)) < \varepsilon/2. \tag{2.11}$$

Putting (2.7), (2.8) and (2.10) together, we get that for $y \in S_C$, $d(x,y) < \delta$ implies

$$\begin{aligned} d(\phi_\infty(x), \phi_\infty(y)) &\leq d(\phi_\infty(x), \phi_t(x)) + d(\phi_t(x), \phi_t(y)) + d(\phi_t(y), \phi_\infty(y)) \\ &< \varepsilon/4 + \varepsilon/4 + \varepsilon/2 = \varepsilon. \end{aligned}$$

This proves that $\phi_\infty \colon S_C \to C$ is continuous.

Finally it follows from the argument above that for any $y_0 \in S_C$ and any $\varepsilon > 0$ there are $\delta > 0$ and $\tau > 0$ so that

$$t > \tau \text{ and } d(y, y_0) < \delta \Rightarrow d(\phi_t(y), \phi_\infty(y_0)) < \varepsilon$$

for all $y \in S_C$. Consequently

$$\phi \colon [0, \infty] \times S_C \to S_C, \quad (t, y) \mapsto \phi_t(y)$$

is continuous. That is, S_C deformation retracts onto C. This concludes the proof of Theorem 1.1. $\square$

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