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Autor: Kappeler, Thomas / Latschev, Janko

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$\operatorname{Ker} T$. Then, using the orthonormal basis $\{f_1, \dots, f_n\}$ of E and the elements $\{h_{k+1}, \dots, h_n\} \in (\operatorname{Ker} T)^{\perp}$ as in Example A.2, we see that

$$\widetilde{\mu}_T = \bigwedge_{i=1}^{k}(e_i, 0, 0) \wedge \bigwedge_{l=k+1}^{n} (e_l, Te_l, 0) \wedge \bigwedge_{j=1}^{n}(0, 0, e_j)$$
$$= c \cdot \bigwedge_{i=1}^{k}(e_i, 0, 0) \wedge \bigwedge_{r=k+1}^{n} (h_r, f_r, 0) \wedge \bigwedge_{s=1}^{n}(0, 0, f_s),$$

where

$$c = \det(\langle Te_l, f_r\rangle_{l,r=k+1}^{n}) \cdot \det(\langle e_j, f_s\rangle_{j,s=1}^{n}).$$

In particular, $k = 0$ in the above formula whenever T is invertible, so that the expression then simplifies to $c = \det(T)$.

To recapitulate the above, we have shown that there is a natural trivialization of $\mathcal{D}et(\mathcal{T})$ determined by the canonical generator of $\mathcal{D}et(\mathcal{T}_0)$, which when evaluated at an invertible map $T \in L(E, E)$ corresponds to the element

$$\mu_T = \det(T)\, 1 \otimes 1 \in \mathcal{D}et(\mathcal{T}_T). \tag{A.8}$$

This explains the name determinant line bundle for the bundle $\mathcal{D}et(\mathcal{T})$ and concludes our discussion of Example A.7.

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Thomas Kappeler

Institut für Mathematik
Universität Zürich
Winterthurerstr. 157
CH-8057 Zürich
Switzerland
e-mail: tk@math.unizh.ch

Janko Latschev

Institut für Mathematik
Humboldt-Universität zu Berlin
Unter den Linden 6
D-10099 Berlin
Germany
e-mail: latschev@math.hu-berlin.de

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