must be equivalent to one of a finite number of forms $r_i h_i$. Now let q be the least common denominator of the coefficients of $r_i h_i$. If we could have $f'(x_1, x_2, \ldots, x_{n-1}, 0)$ equivalent to $r_i h_i$ for an infinite number of possible values of α_2, with $0 \leq \alpha_2 < 1$, there would be two allowable values, say β and γ, with $0 < |\beta - \gamma| < \frac{1}{2q}$. Then considering $f'(0, 1, 0, \ldots, 0)$, we see that $f'(x_1, x_2, \ldots, x_{n-1}, 0)$ represents $k - \beta^2$ for $\alpha_2 = \beta$ and represents $k - \gamma^2$ for $\alpha_2 = \gamma$. However

$$\left|(k - \beta^2) - (k - \gamma^2)\right| = \left|\beta^2 - \gamma^2\right| < \frac{1}{2q}|\beta + \gamma| < \frac{1}{q}$$

contradicting the fact that distinct values of $r_i h_i$ are never closer than $\frac{1}{q}$. Similar considerations of $f'(0, 0, 1, \ldots, 0), \ldots, f'(0, 0, \ldots, 1, 0)$ show that there are only a finite number of allowable values of $\alpha_3, \ldots, \alpha_{n-1}$ for each $r_i h_i$.

Finally, to show that the number of allowable values of α_n in (8) is finite, consider the indefinite $(n-1)$-ary sections $f'(x_1, x_2, \ldots, x_{n-2}, 0, x_n)$, $f'(x_1, x_2, \ldots, x_{n-2}, x_n, x_n)$ and $f'(x_1, x_2, \ldots, x_{n-2}, 2x_n, x_n)$. At least one of these, called $\psi(x_1, x_2, \ldots, x_{n-2}, x_n)$ say, has a non-zero determinant (whose value depends only on k and the coefficients of g'). So, taking $\varepsilon_2 = \varepsilon |D(f)|^{1/n} |D(\psi)|^{-1/n-1} > 0$, ψ will represent a small positive value

$$v_2 < \varepsilon_2 |D(\psi)|^{1/n-1} = \varepsilon |D(f)|^{1/n}$$

unless it is equivalent to a multiple of one of a finite number of forms. Since $N(\psi) = 1$, there will again only be one allowable multiple for each of the finite number of forms. As before, we can also see that for each of the finite number of possibilities for $\alpha_2, \ldots, \alpha_{n-1}, k, g'$ there will only be a finite number of allowable values of α_n.

It follows that the number of forms $f \in \mathcal{E}_n$ for which (6) fails (for a given $\varepsilon > 0$) is finite. So the theorem holds for n–ary forms.

REFERENCES

[1] CASSELS, J. W. S. *An Introduction to Diophantine Approximation*. Camb. Univ. Press, Cambridge, 1957.

[2] —— *An Introduction to the Geometry of Numbers*. Springer, Berlin, 1971.

[3] CASSELS, J. W. S. and H. P. F. SWINNERTON-DYER. On the product of three homogeneous linear forms and indefinite ternary quadratic forms. *Phil. Trans. Roy. Soc. London 248* (1955), 73–96.

[4] DANI, S. G. and G. MARGULIS. Values of quadratic forms at integral points: an elementary approach. *L'Enseign. Math. (2) 36* (1990), 143–174.

[5] DAVENPORT, H. On indefinite ternary quadratic forms. *Proc. London Math. Soc. (2) 51* (1950), 145–160.

[6] JACKSON, T. H. Small positive values of indefinite binary quadratic forms. *J. London Math. Soc. 43* (1968), 730–738.

[7] —— Small positive values of indefinite quadratic forms. *J. London Math. Soc. (2) 1* (1969), 643–659.

[8] OPPENHEIM, A. One-sided inequalities for quadratic forms: II quaternary forms. *Proc. London Math. Soc. (3) 3* (1953), 418–429.

[9] —— Values of quadratic forms (I), (II). *Quart. J. Math. Oxford Ser. (2) 4* (1953), 54–66.

[10] VENKOV, B. A. On the extremal problem of Markov for indefinite ternary quadratic forms. *Izv. Akad. Nauk SSSR 9* (1945), 429–494.

[11] VULAKH, L. Y. On minima of indefinite rational quadratic forms. *J. Number Theory 21* (1985), 275–285.

[12] WATSON, G. L. One-sided inequalities for integral quadratic forms. *Quart. J. Math. Oxford Ser. (2) 9* (1958), 99–108.

[13] —— *Integral Quadratic Forms*. Camb. Univ. Press, Cambridge, 1960.

[14] —— Asymmetric inequalities for indefinite quadratic forms. *Proc. London Math. Soc. (3) 18* (1968), 95–113.

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Leere Seite
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Page vide