

Objektyp: **ReferenceList**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **50 (2004)**

Heft 3-4: **L'enseignement mathématique**

PDF erstellt am: **21.09.2024**

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interpreting the former as elementary symmetric functions in certain roots of  $G$  (or their squares).

Ehresmann [E] investigated the topology of complex Grassmann manifolds (and other hermitian symmetric spaces) by studying the algebra of  $K$ -invariant differential forms on them ( $K = U(N)$  for  $X = G(m, n)$ ). This relies on the fact that the invariant forms are harmonic for the natural hermitian structure on  $X$ , which implies that the ring of all such forms is isomorphic to  $H^*(X)$ . Kostant [K1] [K2] later found analogues of these results for arbitrary (generalized) flag manifolds. The representation theory used to determine the  $K$ -invariant forms in this program does not directly relate the multiplicities  $c_{\lambda\mu}^{\nu}$  in equations (1) and (2). Note however that the cited works of É. Cartan and Ehresmann were used by Chern in his fundamental paper on the characteristic classes of complex manifolds [Ch1]. More recently, Stoll [St] used fiber integration to study the algebra of invariant forms on the Grassmannian, but his work does not address the question posed in the Introduction.

Following [SGA6], [V] and [Be], the isomorphism between  $\mathrm{gr}R(G)$  and  $\mathrm{Pol}(\mathfrak{g})^G$  in §6 may be used to construct the Chern-Weil (or characteristic) homomorphism in algebraic geometry. Let  $P \rightarrow X$  be a principal  $G$ -bundle over a smooth algebraic variety  $X$  and let  $CH^*(X)$  denote the Chow group of algebraic cycles on  $X$  modulo rational equivalence. The Grothendieck group  $K(X)$  of vector bundles on  $X$  is a  $\lambda$ -ring, with the  $\lambda$ -operations induced by exterior powers. According to [SGA6, Exp. XIV], the graded ring  $\mathrm{gr}K(X) \otimes \mathbf{R}$  is canonically isomorphic to the real Chow ring  $CH_{\mathbf{R}}^*(X) = CH^*(X) \otimes \mathbf{R}$ . There is a natural  $\lambda$ -ring homomorphism  $\pi: \mathcal{R}(G) \rightarrow K(X)$ , defined by sending a representation  $G \rightarrow \mathrm{GL}(E)$  to the associated vector bundle  $P \times_G E$  over  $X$ . The characteristic homomorphism is the induced map

$$\mathrm{gr}(\pi)_{\mathbf{R}}: \mathrm{Pol}(\mathfrak{g})^G \longrightarrow CH_{\mathbf{R}}^*(X).$$

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(Reçu le 22 février 2004)

Harry Tamvakis

Department of Mathematics  
Brandeis University – MS 050  
P. O. Box 9110  
Waltham, MA 02454-9110  
U. S. A.  
*e-mail*: harryt@brandeis.edu