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STEP 6. Note that  $\bar{A}$  is also the normalization of  $B$ . We will compute  $\dim(\bar{A}/B)$ .

By [Se], p.59, Formula (3), the question can again be reduced to the complete case. Let  $D$  be the completion of  $B$ . Then

$$D = k[[w, u]] / (\epsilon u^{m-n} - w^n \prod_{1 \leq i \leq m} (1 + \lambda_i u)).$$

Find  $\alpha \in D$  such that  $\alpha^{m-n} = \epsilon^{-1} \prod_{1 \leq i \leq m} (1 + \lambda_i u)$ . Define  $U = u/\alpha$ . Then  $D \simeq k[[w, U]] / (U^{m-n} - w^n)$ . Now we can apply Lemma 4 to get  $\dim(\bar{D}/D) = \{(n-1)(m-n-1) - 1 + d\}/2$ .

STEP 7. Finally we find that  $\delta_p = \dim(\bar{A}/A) = \dim(B/A) + \dim(\bar{A}/B) = \{(m-1)(m-n-1) - 1 + d\}/2$ . Thus

$$(N-1)(N-2)/2 - \delta_p = \{(m-1)(n-1) + 1 - d\}/2$$

because  $N = \max\{m, n\} = m$ . This completes the proof of Theorem 2.

REMARKS.

(1) From the above proof, it is clear that Theorem 2 remains valid if  $\text{char } k = p > 0$  and  $p$  doesn't divide  $mn \prod_{1 \leq i \leq l} m_i n_i$ .

(2) Similarly, if  $p$  is a prime number and the affine curve is defined by  $y^p = \prod_{1 \leq i \leq l} (x - \lambda_i)^{m_i}$  such that the  $\lambda_i$  are distinct,  $1 \leq m_i < p$  and  $p$  doesn't divide  $\sum_{1 \leq i \leq l} m_i$ , then Theorem 2 (and its proof for this case) remains valid no matter what  $\text{char } k$  may be. Note that the latter assumption can always be achieved. For, if we denote  $\sum_{1 \leq i \leq l} m_i$  by  $m$  and suppose that  $m = pr$ , we may assume that  $\lambda_1 = 0$ . Divide both sides of the equation by  $x^m$ . Consider the new variables  $u = 1/x, v = y/x^r$ .

(3) On the other hand, if we assume that  $k$  is a perfect field (such that (i)  $p \nmid mn \prod_{1 \leq i \leq l} m_i n_i$  if  $\text{char } k = p > 0$ , or (ii)  $p$  is a prime number and the affine curve is defined by  $y^p = \prod_{1 \leq i \leq l} (x - \lambda_i)^{m_i}$  with ...) but not algebraically closed, then Theorem 2 is true because we can extend the constant field  $k$  to its algebraic closure at the beginning of the proof without affecting the genus by [Ch], p. 99.

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