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## A GENUS FORMULA FOR SOME PLANE CURVES

by Ming-Chang KANG<sup>\*)</sup>

**ABSTRACT.** Let  $k$  be an algebraically closed field with  $\text{char } k = 0$ , and  $\Gamma_0$  be the irreducible affine curve defined by the equation  $f(x) = g(y)$ . We determine the genus of  $\Gamma_0$  in terms of the numerical data of  $f$  and  $g$ .

### 1. INTRODUCTION

Throughout this article  $k$  is an algebraically closed field with  $\text{char } k = 0$ , unless otherwise specified. (See the remarks at the end of this note for weaker assumptions about the field  $k$ .)

Let  $f(T), g(T) \in k[T]$  be non-constant monic polynomials, let  $\Gamma_0$  be the affine plane curve defined by the equation  $f(x) = g(y)$ , and let  $\Gamma$  be the projective plane curve associated to  $\Gamma_0$ , i.e.  $\Gamma$  is defined by  $X_0^N f(X_1/X_0) = X_0^N g(X_2/X_0)$ , where  $N = \max\{\deg f, \deg g\}$ ,  $x = X_1/X_0$ ,  $y = X_2/X_0$ . Assume that  $f(x) - g(y) \in k[x, y]$  is an irreducible polynomial. The purpose of this note is to find the genus of (the normalization of) the plane algebraic curve  $\Gamma$  in terms of the numerical data of  $f(x)$  and  $g(y)$ . The class of algebraic curves  $\Gamma_0$  of the form  $f(x) = g(y)$  includes hyper-elliptic curves, Fermat curves, some curves arising in arithmetic questions and coding theory, etc. (see Theorem 1 and [Pr]). It is desirable to find an explicit formula for the genus of such a plane curve.

Let  $m = \deg f$ ,  $n = \deg g$ ,  $d = \gcd\{m, n\}$ . Define  $R = \{a \in k : f'(a) = 0\}$ ,  $S = \{b \in k : g'(b) = 0\}$ ,  $\text{Sing}(\Gamma_0) = \{(a, b) \in R \times S : f(a) = g(b)\}$ . We list all the elements of  $\text{Sing}(\Gamma_0)$  as  $(a_1, b_1), (a_2, b_2), \dots, (a_l, b_l)$ . (It may happen that  $l = 0$ , i.e. that  $\text{Sing}(\Gamma_0) = \emptyset$ .) For each  $(a_i, b_i) \in \text{Sing}(\Gamma_0)$ , write

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$f(x) - g(y) = (x - a_i)^{m_i} f_1(x) - (y - b_i)^{n_i} g_1(y)$ , where  $f_1(a_i) g_1(b_i) \neq 0$ . Define  $d_i = \gcd\{m_i, n_i\}$ .

The genus of the curve  $\Gamma_0$  was estimated by Davenport, Lewis and Schinzel in connection with integral solutions of  $f(x) = g(y)$ , viz.

**THEOREM 1** ([DLS], Theorem 1). *Assume that  $f(T), g(T) \in \mathbf{Z}[T]$  and that there are distinct values  $a_1, a_2, \dots, a_t \in R$  such that  $t \geq [m/2]$  with  $f(x) - g(y) = (x - a_i)^{m_i} f_1(x) - h_i(y)$ , where  $f_1(a_i) \neq 0$  and  $h_i(y) = 0$  has no multiple roots for  $1 \leq i \leq t$ . Then  $f(x) - g(y) \in \mathbf{C}[x, y]$  is irreducible. Moreover, if  $n \neq 2$  and  $(m, n) \neq (3, 3)$ , then the genus of  $\Gamma_0$  is positive and the equation  $f(x) = g(y)$  has finitely many integral solutions.*

The main result of this article is the following

**THEOREM 2.** *Let  $f(T), g(T) \in k[T]$  and let  $\Gamma_0, \Gamma, m, n, d, m_i, n_i, d_i$  be defined as above. Assume that  $\Gamma_0$  is an irreducible curve. Then the genus of  $\Gamma$  is equal to*

$$\{(m-1)(n-1) + 1 - d\}/2 - \sum_{1 \leq i \leq l} \{(m_i-1)(n_i-1) - 1 + d_i\}/2.$$

Note that, if  $\Gamma$  is a non-singular curve, it is necessary that  $|m - n| \leq 1$ , i.e. all the points of  $\Gamma$  lying on the infinite line  $X_0 = 0$  are non-singular points. If  $\Gamma$  is non-singular and  $m = n$ , the above formula reduces to the well-known formula  $(m-1)(m-2)/2$ ; if  $\Gamma$  is non-singular and  $|m - n| = 1$ , the genus of  $\Gamma$  becomes  $(m-1)(n-1)/2$ .

In [Mi], p. 74 (Problem III.2 D) it was suggested to find a genus formula for the cyclic covering  $y^n = f(x)$  of the affine line by applying Hurwitz's formula, although no explicit formula was exhibited there. Instead we will prove Theorem 2 by using Plücker's formula (see Theorem 3 in Section 2).

We emphasize that in Theorem 2 one must assume that the curve  $f(x) - g(y) = 0$  is irreducible. This assumption is not a very serious restriction. For, as a consequence of the classification of finite simple groups, if  $f(T), g(T)$  are indecomposable polynomials, then  $f(x) - g(y)$  is irreducible except when (i)  $g(T) = f(aT + b)$  for some  $a, b \in k$ , or (ii)  $\deg f = \deg g = 7, 11, 13, 15, 21$ , or 31. (See [Fe], Theorem 1.1. A polynomial  $f(T) \in k[T]$  is *indecomposable* if, whenever  $f(T) = f_1(f_2(T))$  for polynomials  $f_1(T), f_2(T) \in k[T]$ , we have  $\deg f_i = 1$  for  $i = 1$  or 2.)

## 2. THE PROOF

We recall Plücker's formula first. (See [Se], p. 65, Formula (19); [Hi]; [Ca], pp. 111–113. See also the remark in [BK], Theorem 5, p. 614.)

**THEOREM 3.** *Let  $\Gamma$  be a projective plane curve of degree  $N$ . Then the genus of  $\Gamma$  is equal to*

$$(N-1)(N-2)/2 - \sum \delta_P,$$

where  $P$  runs over the singular points of  $\Gamma$ ,  $\mathcal{O}_P$  denotes the local ring at  $P$ ,  $\bar{\mathcal{O}}_P$  is the normalization of  $\mathcal{O}_P$ , and  $\delta_P = \dim(\bar{\mathcal{O}}_P/\mathcal{O}_P)$ .

**LEMMA 4.** *Let  $k[[x, y]]$  be the power series ring,  $m$  and  $n$  be positive integers,  $A = k[[x, y]]/(x^m - y^n)$ , and  $\bar{A}$  the normalization of  $A$ . If  $d = \gcd\{m, n\}$ , then  $\dim_k(\bar{A}/A) = \{(m-1)(n-1) - 1 + d\}/2$ .*

*Proof.* STEP 1. If  $d = \gcd\{m, n\} = 1$ , then  $\dim(\bar{A}/A) = (m-1)(n-1)/2$ .

Note that  $A = k[[x, y]]/(x^m - y^n) \simeq k[[t^n, t^m]] \hookrightarrow k[[t]] \simeq \bar{A}$ . Since  $A$  is a complete intersection,  $A$  is a Gorenstein local ring and we can apply [Se], p. 72, Proposition 7. It follows that  $\dim(\bar{A}/A) = n_p/2$ , where  $\langle t^{n_p} \rangle$  is the conductor ideal of  $\bar{A} \cong k[[t]]$  into  $A \cong k[[t^n, t^m]]$ .

On the other hand, the conductor ideal  $\langle t^M \rangle$  of  $k[[t]]$  into  $k[[t^n, t^m]]$  is simply that given by  $M = \min\{p \in \mathbf{N} : \text{for any } q \geq p, q \text{ can be written as } nx + my \text{ for some non-negative integers } x, y\}$ . It is not difficult to determine this non-negative integer  $M$ . In fact,  $M = (m-1)(n-1)$ . (See [NZ], p. 107, Exercise 9.) Hence the result.

STEP 2. Now consider the case  $d = \gcd\{m, n\} \geq 2$ . Write  $m = dr, n = ds$ .

Let  $\zeta$  be a primitive  $d$ -th root of unity. Note that  $x^m - y^n = (x^r)^d - (y^s)^d = \prod_{1 \leq i \leq d} (x^r - \zeta^i y^s)$ . All factors  $x^r - \zeta^i y^s$  are relatively prime irreducible elements in  $k[[x, y]]$  because  $k[[x, y]]/(x^r - \zeta^i y^s) \cong k[[t^s, t^r]]$  is a subring of  $k[[t]]$ . (Note that the factor  $\zeta^i$  in  $x^r - \zeta^i y^s$  can be absorbed into  $y^s$ .)

Let  $I_i$  be the prime ideal in  $k[[x, y]]$  generated by  $x^r - \zeta^i y^s$ . Since  $I_1 I_2 \cdots I_d = I_1 \cap I_2 \cap \cdots \cap I_d$ , it follows that

$$A = k[[x, y]] / \prod_{1 \leq i \leq d} (x^r - \zeta^i y^s) = k[[x, y]] / I_1 \cdots I_d \hookrightarrow \prod_{1 \leq j \leq d} B_j,$$

where  $B_j = k[[x, y]]/I_j$ .

On the other hand, the set of non-zero divisors of  $A = k[[x, y]]/I_1 \cap \cdots \cap I_d$  is the image of  $S = k[[x, y]] \setminus I_1 \cup \cdots \cup I_d$ . Thus the total quotient ring of  $A$  is  $S^{-1}A$ , which is just the total quotient ring of  $\prod_{1 \leq j \leq d} B_j$ . It follows that the normalization of  $A$  is  $\prod_{1 \leq j \leq d} \bar{B}_j$ , where  $\bar{B}_j$  is the normalization of  $B_j$ .

To sum up,  $\dim(\bar{A}/A) = \dim(\prod_{1 \leq j \leq d} B_j/A) + \sum_{1 \leq j \leq d} \dim(\bar{B}_j/B_j)$ . Note that  $\dim(\bar{B}_j/B_j) = (r-1)(s-1)/2$  by Step 1 since  $\gcd\{r, s\} = 1$ . It remains to prove that  $\dim(\prod_{1 \leq j \leq d} B_j/A) = d(d-1)rs/2$ .

STEP 3. We will prove that  $\dim(\prod_{1 \leq j \leq d} B_j / A) = d(d-1)rs/2$ .

Define  $C_j = k[[x, y]]/I_j \cap I_{j+1} \cap \cdots \cap I_d$  and  $D_j = k[[x, y]]/(I_{j-1}, I_j \cap \cdots \cap I_d)$  for  $2 \leq j \leq d$ . We get the following short exact sequences

It follows that  $A \subset B_1 \times C_2 \subset B_1 \times B_2 \times C_3 \subset \cdots \subset B_1 \times B_2 \times \cdots \times B_d$  since  $C_d = B_d$ . Hence

$$\dim\left(\prod_{1 \leq j \leq d} B_j/A\right) = \dim\left((B_1 \times C_2)/A\right) + \sum_{3 \leq j \leq d} \dim((B_{j-1} \times C_j)/C_{j-1}) = \sum_{2 \leq j \leq d} \dim(D_j).$$

Note that

$$D_j = k[[x, y]]/(I_{j-1}, I_j \cap \cdots \cap I_d) = k[[x, y]]/\left(x^r - \zeta^{j-1} y^s, \prod_{j \leq i \leq d} (x^r - \zeta^i y^s)\right) \\ = k[[x, y]]/(x^r - \zeta^{j-1} y^s, y^{s(d-j+1)}).$$

Hence  $\dim(D_j) = rs(d-j+1)$ . Thus  $\sum_{2 \leq j \leq d} \dim(D_j) = d(d-1)rs/2$ .  $\square$

## PROOF OF THEOREM 2

The singular points  $P$  of  $\Gamma$  are either points belonging to  $\text{Sing}(\Gamma_0)$  or points lying on the infinite line  $X_0 = 0$ . We shall compute  $\delta_P$  and apply Theorem 3.

STEP 1.  $P = (a_i, b_i) \in \text{Sing}(\Gamma_0)$ .

Let  $\delta_i = \dim(\bar{R}_i/R_i)$  where  $R_i$  is the local ring at  $(a_i, b_i)$  and  $\bar{R}_i$  the normalization of  $R_i$ , for  $1 \leq i \leq l$ .

Since  $\delta_i$  is invariant under completion by [Se], p. 59, Formula (3), it suffices to compute  $\delta_i = \dim(\bar{A}_i/A_i)$ , where  $A_i \simeq k[[x, y]]/(x^{m_i} - y^{n_i})$  is the completion of  $R_i$  and  $\bar{A}_i$  is the normalization of  $A_i$ . (For, considering the case  $i = 1$ , we may assume that  $a_1 = b_1 = 0$  and write  $f(x) = x^{m_1}f_1(x)$ ,  $g(y) = y^{n_1}g_1(y)$  where  $f_1(0)g_1(0) \neq 0$ . The elements  $f_1(x)$ ,  $g_1(y)$  are units in  $k[[x, y]]$  and can be written as  $\beta^{m_1}$ ,  $\gamma^{n_1}$  for some  $\beta, \gamma \in k[[x, y]]$ . Define  $X = x\beta$  and  $Y = y\gamma$ . Then  $A_1 \simeq k[[x, y]]/(g(y) - f(x)) \simeq k[[X, Y]]/(Y^{n_1} - X^{m_1})$ .) By Lemma 4, it follows that  $\dim(\bar{A}_i/A_i) = \{(m_i - 1)(n_i - 1) - 1 + d_i\}/2$ .

STEP 2. If  $|m - n| \leq 1$ , then the projective curve is non-singular except for those points belonging to  $\text{Sing}(\Gamma_0)$ . It is easy to check that  $(N - 1)(N - 2)/2 = \{(m - 1)(n - 1) + 1 - d\}/2$ , where  $N = \max\{m, n\}$ .

STEP 3. Consider the case  $m \geq n + 2$ . (The case  $m \leq n - 2$  is similar and will be omitted.)

Consider the homogenized polynomial equation  $X_0^m g(X_2/X_0) = X_0^m f(X_1/X_0)$  where  $x = X_1/X_0$ ,  $y = X_2/X_0$  and we shall write  $f(x) = \prod_{1 \leq i \leq m} (x + \lambda_i)$ ,  $g(y) = \prod_{1 \leq j \leq n} (y + \rho_j)$ . The only singular point of  $\Gamma$  other than those belonging to  $\text{Sing}(\Gamma_0)$  is  $P = (0 : 0 : 1)$ . Let  $z = X_0/X_2$ ,  $w = X_1/X_2$ . The dehomogenized polynomial becomes  $z^{m-n} \prod_{1 \leq j \leq n} (1 + \rho_j z) = \prod_{1 \leq i \leq m} (w + \lambda_i z)$ . It suffices to compute  $\delta_P = \dim(\bar{A}/A)$ , where

$$A = k[w, z]_{(w, z)} / \left( z^{m-n} \prod_{1 \leq j \leq n} (1 + \rho_j z) - \prod_{1 \leq i \leq m} (w + \lambda_i z) \right),$$

the local ring of  $\Gamma$  at the point  $P$ . Note that the multiplicity at the point  $P$  is  $m - n$ .

STEP 4. The element  $\prod_{1 \leq j \leq n} (1 + \rho_j z)$  is a unit in  $A$ . Call it  $\epsilon$ .

In the local ring  $A$ , consider the relation:  $\epsilon z^{m-n} = \prod_{1 \leq i \leq m} (w + \lambda_i z)$ . Define  $u = z/w$  in the quotient field of  $A$ . The above relation becomes  $\epsilon u^{m-n} = w^n \prod_{1 \leq i \leq m} (1 + \lambda_i u)$ .

Write  $\prod_{1 \leq i \leq m} (1 + \lambda_i u) = \sum_{0 \leq i \leq m} a_i u^i$ , where  $a_i \in k$  and  $a_0 = 1$ . Then we get  $\epsilon u^{m-n} - w^n \sum_{m-n \leq i \leq m} a_i u^i = w^n \sum_{0 \leq i \leq m-n-1} a_i u^i$ . Hence

$$u^{m-n} (\epsilon - \sum_{m-n \leq i \leq m} a_i w^n u^{i-m+n}) = \sum_{0 \leq i \leq m-n-1} a_i w^n u^i.$$

As

$$\epsilon - \sum_{m-n \leq i \leq m} a_i w^n u^{i-m+n} = \epsilon - a_{m-n} w^n - a_{m-n+1} w^{n-1} z - \cdots - a_m z^m$$

is a unit in  $A$ , we find

$$(1) \quad u^{m-n} + \alpha_1 u^{m-n-1} + \alpha_2 u^{m-n-2} + \cdots + \alpha_{m-n} = 0,$$

where  $\alpha_i \in w^n A$ . It follows that  $u \in \bar{A}$ .

Clearly,  $A \subset B \subset \bar{A}$ , where

$$B = A[u] = k[w, u]_{(w, u)} / (\epsilon u^{m-n} - w^n \prod_{1 \leq i \leq m} (1 + \lambda_i u)).$$

Thus  $\delta_P = \dim(\bar{A}/A) = \dim(B/A) + \dim(\bar{A}/B)$ .

STEP 5. We claim that  $\dim(B/A) = (m-n)(m-n-1)/2$ .

First we will show that

$$(2) \quad B = A + \sum_{0 \leq i < j \leq m-n-1} k \cdot w^i u^j.$$

Let  $C$  be the completion of  $A$ . Then  $C = k[[w, z]] / (\epsilon z^{m-n} - \prod_{1 \leq i \leq m} (w + \lambda_i z))$ . Since  $B/A = A[u]/A$  is a finite-dimensional vector space over  $k$ , it is naturally isomorphic to  $(A[u]/A) \otimes_A C$ . Thus (2) is equivalent to

$$(3) \quad C[u] = C + \sum_{0 \leq i < j \leq m-n-1} k \cdot w^i u^j.$$

To check the validity of (3) it suffices to consider whether  $z^i u^j$  (where  $0 \leq j \leq m-n-1$ ) belongs to the right-hand-side of (3), because of Formula (1) and the relation  $uw = z$ . We will prove it by induction on  $j$ . If  $j = 0$ , it is trivial. Now assume  $j \geq 1$ . In case  $j \leq m-n-i-1$ , then  $z^i u^j = w^i u^{i+j}$  with  $i+j \leq m-n-1$  as required. If  $j \geq m-n-i$ , then  $z^i u^j = w^i u^{i+j}$  and  $i+j \geq m-n$ . Using (1) we find that  $u^{i+j}$  is a linear combination of  $w^n u^l$  with coefficients in  $A$ , where  $0 \leq l \leq m-n-1$ . It follows that, after modulo  $C$ ,  $z^i u^j$  is a linear combination of  $u^{l-n-i}$  (if  $l-n-i > 0$ ) where  $l \leq m-n-1$ . Note that  $l-n-i \leq m-2n-i-1 < j$ . Thus we have reduced  $z^i u^j$  to terms of lower exponents.

We shall show that the  $w^i u^j$  are linearly independent in  $B/A$  where  $0 \leq i < j \leq m-n-1$ . Suppose  $0 = \sum_{i < j} a_{ij} w^i u^j$  in  $B/A$  where  $a_{ij} \in k$  are not all zero. Hence  $\sum_{i < j} a_{ij} w^i u^j = \varphi \in A$ . Define  $p = \max\{j-i : a_{ij} \neq 0\}$ . Multiply the relation  $\sum_{i < j} a_{ij} w^i u^j = \varphi$  by  $w^p$ . We get  $\sum_{j-i=p} a_{ij} z^j = w\psi$  for some  $\psi \in A$ . Since  $1 \leq j \leq m-n-1$ , it follows that the multiplicity of the point  $P$  is  $\leq m-n-1$ , a contradiction.

STEP 6. Note that  $\bar{A}$  is also the normalization of  $B$ . We will compute  $\dim(\bar{A}/B)$ .

By [Se], p. 59, Formula (3), the question can again be reduced to the complete case. Let  $D$  be the completion of  $B$ . Then

$$D = k[[w, u]] / (\epsilon u^{m-n} - w^n \prod_{1 \leq i \leq m} (1 + \lambda_i u)).$$

Find  $\alpha \in D$  such that  $\alpha^{m-n} = \epsilon^{-1} \prod_{1 \leq i \leq m} (1 + \lambda_i u)$ . Define  $U = u/\alpha$ . Then  $D \simeq k[[w, U]] / (U^{m-n} - w^n)$ . Now we can apply Lemma 4 to get  $\dim(\bar{D}/D) = \{(n-1)(m-n-1) - 1 + d\}/2$ .

STEP 7. Finally we find that  $\delta_P = \dim(\bar{A}/A) = \dim(B/A) + \dim(\bar{A}/B) = \{(m-1)(m-n-1) - 1 + d\}/2$ . Thus

$$(N-1)(N-2)/2 - \delta_P = \{(m-1)(n-1) + 1 - d\}/2$$

because  $N = \max\{m, n\} = m$ . This completes the proof of Theorem 2.

#### REMARKS.

(1) From the above proof, it is clear that Theorem 2 remains valid if  $\text{char } k = p > 0$  and  $p$  doesn't divide  $mn \prod_{1 \leq i \leq l} m_i n_i$ .

(2) Similarly, if  $p$  is a prime number and the affine curve is defined by  $y^p = \prod_{1 \leq i \leq l} (x - \lambda_i)^{m_i}$  such that the  $\lambda_i$  are distinct,  $1 \leq m_i < p$  and  $p$  doesn't divide  $\sum_{1 \leq i \leq l} m_i$ , then Theorem 2 (and its proof for this case) remains valid no matter what  $\text{char } k$  may be. Note that the latter assumption can always be achieved. For, if we denote  $\sum_{1 \leq i \leq l} m_i$  by  $m$  and suppose that  $m = pr$ , we may assume that  $\lambda_1 = 0$ . Divide both sides of the equation by  $x^n$ . Consider the new variables  $u = 1/x$ ,  $v = y/x^r$ .

(3) On the other hand, if we assume that  $k$  is a perfect field (such that (i)  $p \nmid mn \prod_{1 \leq i \leq l} m_i n_i$  if  $\text{char } k = p > 0$ , or (ii)  $p$  is a prime number and the affine curve is defined by  $y^p = \prod_{1 \leq i \leq l} (x - \lambda_i)^{m_i}$  with ...) but not algebraically closed, then Theorem 2 is true because we can extend the constant field  $k$  to its algebraic closure at the beginning of the proof without affecting the genus by [Ch], p. 99.

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