

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 50 (2004)  
**Heft:** 1-2: L'enseignement mathématique

**Artikel:** A genus formula for some plane curves  
**Autor:** Kang, Ming-Chang  
**DOI:** <https://doi.org/10.5169/seals-2644>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 07.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## A GENUS FORMULA FOR SOME PLANE CURVES

by Ming-Chang KANG<sup>\*</sup>)

ABSTRACT. Let  $k$  be an algebraically closed field with  $\text{char } k = 0$ , and  $\Gamma_0$  be the irreducible affine curve defined by the equation  $f(x) = g(y)$ . We determine the genus of  $\Gamma_0$  in terms of the numerical data of  $f$  and  $g$ .

### 1. INTRODUCTION

Throughout this article  $k$  is an algebraically closed field with  $\text{char } k = 0$ , unless otherwise specified. (See the remarks at the end of this note for weaker assumptions about the field  $k$ .)

Let  $f(T), g(T) \in k[T]$  be non-constant monic polynomials, let  $\Gamma_0$  be the affine plane curve defined by the equation  $f(x) = g(y)$ , and let  $\Gamma$  be the projective plane curve associated to  $\Gamma_0$ , i.e.  $\Gamma$  is defined by  $X_0^N f(X_1/X_0) = X_0^N g(X_2/X_0)$ , where  $N = \max\{\deg f, \deg g\}$ ,  $x = X_1/X_0$ ,  $y = X_2/X_0$ . Assume that  $f(x) - g(y) \in k[x, y]$  is an irreducible polynomial. The purpose of this note is to find the genus of (the normalization of) the plane algebraic curve  $\Gamma$  in terms of the numerical data of  $f(x)$  and  $g(y)$ . The class of algebraic curves  $\Gamma_0$  of the form  $f(x) = g(y)$  includes hyper-elliptic curves, Fermat curves, some curves arising in arithmetic questions and coding theory, etc. (see Theorem 1 and [Pr]). It is desirable to find an explicit formula for the genus of such a plane curve.

Let  $m = \deg f$ ,  $n = \deg g$ ,  $d = \gcd\{m, n\}$ . Define  $R = \{a \in k : f'(a) = 0\}$ ,  $S = \{b \in k : g'(b) = 0\}$ ,  $\text{Sing}(\Gamma_0) = \{(a, b) \in R \times S : f(a) = g(b)\}$ . We list all the elements of  $\text{Sing}(\Gamma_0)$  as  $(a_1, b_1), (a_2, b_2), \dots, (a_l, b_l)$ . (It may happen that  $l = 0$ , i.e. that  $\text{Sing}(\Gamma_0) = \emptyset$ .) For each  $(a_i, b_i) \in \text{Sing}(\Gamma_0)$ , write

---

<sup>\*</sup>) Partially supported by the National Science Council, Republic of China.

$f(x) - g(y) = (x - a_i)^{m_i} f_1(x) - (y - b_i)^{n_i} g_1(y)$ , where  $f_1(a_i) g_1(b_i) \neq 0$ . Define  $d_i = \gcd\{m_i, n_i\}$ .

The genus of the curve  $\Gamma_0$  was estimated by Davenport, Lewis and Schinzel in connection with integral solutions of  $f(x) = g(y)$ , viz.

**THEOREM 1** ([DLS], Theorem 1). *Assume that  $f(T), g(T) \in \mathbf{Z}[T]$  and that there are distinct values  $a_1, a_2, \dots, a_t \in R$  such that  $t \geq [m/2]$  with  $f(x) - g(y) = (x - a_i)^{m_i} f_1(x) - h_i(y)$ , where  $f_1(a_i) \neq 0$  and  $h_i(y) = 0$  has no multiple roots for  $1 \leq i \leq t$ . Then  $f(x) - g(y) \in \mathbf{C}[x, y]$  is irreducible. Moreover, if  $n \neq 2$  and  $(m, n) \neq (3, 3)$ , then the genus of  $\Gamma_0$  is positive and the equation  $f(x) = g(y)$  has finitely many integral solutions.*

The main result of this article is the following

**THEOREM 2.** *Let  $f(T), g(T) \in k[T]$  and let  $\Gamma_0, \Gamma, m, n, d, m_i, n_i, d_i$  be defined as above. Assume that  $\Gamma_0$  is an irreducible curve. Then the genus of  $\Gamma$  is equal to*

$$\{(m-1)(n-1) + 1 - d\}/2 - \sum_{1 \leq i \leq t} \{(m_i-1)(n_i-1) - 1 + d_i\}/2.$$

Note that, if  $\Gamma$  is a non-singular curve, it is necessary that  $|m-n| \leq 1$ , i.e. all the points of  $\Gamma$  lying on the infinite line  $X_0 = 0$  are non-singular points. If  $\Gamma$  is non-singular and  $m = n$ , the above formula reduces to the well-known formula  $(m-1)(m-2)/2$ ; if  $\Gamma$  is non-singular and  $|m-n| = 1$ , the genus of  $\Gamma$  becomes  $(m-1)(n-1)/2$ .

In [Mi], p. 74 (Problem III.2D) it was suggested to find a genus formula for the cyclic covering  $y^n = f(x)$  of the affine line by applying Hurwitz's formula, although no explicit formula was exhibited there. Instead we will prove Theorem 2 by using Plücker's formula (see Theorem 3 in Section 2).

We emphasize that in Theorem 2 one must assume that the curve  $f(x) - g(y) = 0$  is irreducible. This assumption is not a very serious restriction. For, as a consequence of the classification of finite simple groups, if  $f(T), g(T)$  are indecomposable polynomials, then  $f(x) - g(y)$  is irreducible except when (i)  $g(T) = f(aT + b)$  for some  $a, b \in k$ , or (ii)  $\deg f = \deg g = 7, 11, 13, 15, 21$ , or  $31$ . (See [Fe], Theorem 1.1. A polynomial  $f(T) \in k[T]$  is *indecomposable* if, whenever  $f(T) = f_1(f_2(T))$  for polynomials  $f_1(T), f_2(T) \in k[T]$ , we have  $\deg f_i = 1$  for  $i = 1$  or  $2$ .)

2. THE PROOF

We recall Plücker’s formula first. (See [Se], p.65, Formula (19); [Hi]; [Ca], pp.111–113. See also the remark in [BK], Theorem 5, p.614.)

THEOREM 3. *Let  $\Gamma$  be a projective plane curve of degree  $N$ . Then the genus of  $\Gamma$  is equal to*

$$(N - 1)(N - 2)/2 - \sum \delta_P,$$

where  $P$  runs over the singular points of  $\Gamma$ ,  $\mathcal{O}_P$  denotes the local ring at  $P$ ,  $\overline{\mathcal{O}}_P$  is the normalization of  $\mathcal{O}_P$ , and  $\delta_P = \dim(\overline{\mathcal{O}}_P/\mathcal{O}_P)$ .

LEMMA 4. *Let  $k[[x, y]]$  be the power series ring,  $m$  and  $n$  be positive integers,  $A = k[[x, y]]/(x^m - y^n)$ , and  $\bar{A}$  the normalization of  $A$ . If  $d = \gcd\{m, n\}$ , then  $\dim_k(\bar{A}/A) = \{(m - 1)(n - 1) - 1 + d\}/2$ .*

*Proof.* STEP 1. If  $d = \gcd\{m, n\} = 1$ , then  $\dim(\bar{A}/A) = (m - 1)(n - 1)/2$ .

Note that  $A = k[[x, y]]/(x^m - y^n) \simeq k[[t^n, t^m]] \hookrightarrow k[[t]] \simeq \bar{A}$ . Since  $A$  is a complete intersection,  $A$  is a Gorenstein local ring and we can apply [Se], p.72, Proposition 7. It follows that  $\dim(\bar{A}/A) = n_P/2$ , where  $\langle t^{n_P} \rangle$  is the conductor ideal of  $\bar{A} \cong k[[t]]$  into  $A \cong k[[t^n, t^m]]$ .

On the other hand, the conductor ideal  $\langle t^M \rangle$  of  $k[[t]]$  into  $k[[t^n, t^m]]$  is simply that given by  $M = \min\{p \in \mathbf{N} : \text{for any } q \geq p, q \text{ can be written as } nx + my \text{ for some non-negative integers } x, y\}$ . It is not difficult to determine this non-negative integer  $M$ . In fact,  $M = (m - 1)(n - 1)$ . (See [NZ], p.107, Exercise 9.) Hence the result.

STEP 2. Now consider the case  $d = \gcd\{m, n\} \geq 2$ . Write  $m = dr, n = ds$ .

Let  $\zeta$  be a primitive  $d$ -th root of unity. Note that  $x^m - y^n = (x^r)^d - (y^s)^d = \prod_{1 \leq i \leq d} (x^r - \zeta^i y^s)$ . All factors  $x^r - \zeta^i y^s$  are relatively prime irreducible elements in  $k[[x, y]]$  because  $k[[x, y]]/(x^r - \zeta^i y^s) \cong k[[t^s, t^r]]$  is a subring of  $k[[t]]$ . (Note that the factor  $\zeta^i$  in  $x^r - \zeta^i y^s$  can be absorbed into  $y^s$ .)

Let  $I_i$  be the prime ideal in  $k[[x, y]]$  generated by  $x^r - \zeta^i y^s$ . Since  $I_1 I_2 \cdots I_d = I_1 \cap I_2 \cap \cdots \cap I_d$ , it follows that

$$A = k[[x, y]] / \prod_{1 \leq i \leq d} (x^r - \zeta^i y^s) = k[[x, y]] / I_1 \cdots I_d \hookrightarrow \prod_{1 \leq j \leq d} B_j,$$

where  $B_j = k[[x, y]]/I_j$ .

On the other hand, the set of non-zero divisors of  $A = k[[x, y]]/I_1 \cap \cdots \cap I_d$  is the image of  $S = k[[x, y]] \setminus I_1 \cup \cdots \cup I_d$ . Thus the total quotient ring of  $A$  is  $S^{-1}A$ , which is just the total quotient ring of  $\prod_{1 \leq j \leq d} B_j$ . It follows that the normalization of  $A$  is  $\prod_{1 \leq j \leq d} \bar{B}_j$ , where  $\bar{B}_j$  is the normalization of  $B_j$ .

To sum up,  $\dim(\bar{A}/A) = \dim(\prod_{1 \leq j \leq d} B_j/A) + \sum_{1 \leq j \leq d} \dim(\bar{B}_j/B_j)$ . Note that  $\dim(\bar{B}_j/B_j) = (r-1)(s-1)/2$  by Step 1 since  $\gcd\{r, s\} = 1$ . It remains to prove that  $\dim(\prod_{1 \leq j \leq d} B_j/A) = d(d-1)rs/2$ .

STEP 3. We will prove that  $\dim(\prod_{1 \leq j \leq d} B_j/A) = d(d-1)rs/2$ .

Define  $C_j = k[[x, y]]/I_j \cap I_{j+1} \cap \cdots \cap I_d$  and  $D_j = k[[x, y]]/(I_{j-1}, I_j \cap \cdots \cap I_d)$  for  $2 \leq j \leq d$ . We get the following short exact sequences

$$\begin{aligned} 0 &\longrightarrow A &\longrightarrow B_1 \times C_2 &\longrightarrow D_2 &\longrightarrow 0, \\ 0 &\longrightarrow C_2 &\longrightarrow B_2 \times C_3 &\longrightarrow D_3 &\longrightarrow 0, \\ & & \dots & & \\ 0 &\longrightarrow C_{d-1} &\longrightarrow B_{d-1} \times C_d &\longrightarrow D_d &\longrightarrow 0. \end{aligned}$$

It follows that  $A \subset B_1 \times C_2 \subset B_1 \times B_2 \times C_3 \subset \cdots \subset B_1 \times B_2 \times \cdots \times B_d$  since  $C_d = B_d$ . Hence

$$\begin{aligned} \dim\left(\prod_{1 \leq j \leq d} B_j/A\right) &= \dim((B_1 \times C_2)/A) \\ &+ \sum_{3 \leq j \leq d} \dim((B_{j-1} \times C_j)/C_{j-1}) = \sum_{2 \leq j \leq d} \dim(D_j). \end{aligned}$$

Note that

$$\begin{aligned} D_j &= k[[x, y]]/(I_{j-1}, I_j \cap \cdots \cap I_d) = k[[x, y]]/(x^r - \zeta^{j-1}y^s, \prod_{j \leq i \leq d} (x^r - \zeta^i y^s)) \\ &= k[[x, y]]/(x^r - \zeta^{j-1}y^s, y^{s(d-j+1)}). \end{aligned}$$

Hence  $\dim(D_j) = rs(d-j+1)$ . Thus  $\sum_{2 \leq j \leq d} \dim(D_j) = d(d-1)rs/2$ .  $\square$

PROOF OF THEOREM 2

The singular points  $P$  of  $\Gamma$  are either points belonging to  $\text{Sing}(\Gamma_0)$  or points lying on the infinite line  $X_0 = 0$ . We shall compute  $\delta_P$  and apply Theorem 3.

STEP 1.  $P = (a_i, b_i) \in \text{Sing}(\Gamma_0)$ .

Let  $\delta_i = \dim(\bar{R}_i/R_i)$  where  $R_i$  is the local ring at  $(a_i, b_i)$  and  $\bar{R}_i$  the normalization of  $R_i$ , for  $1 \leq i \leq l$ .

Since  $\delta_i$  is invariant under completion by [Se], p. 59, Formula (3), it suffices to compute  $\delta_i = \dim(\bar{A}_i/A_i)$ , where  $A_i \simeq k[[x, y]]/(x^{m_i} - y^{n_i})$  is the completion of  $R_i$  and  $\bar{A}_i$  is the normalization of  $A_i$ . (For, considering the case  $i = 1$ , we may assume that  $a_1 = b_1 = 0$  and write  $f(x) = x^{m_1}f_1(x)$ ,  $g(y) = y^{n_1}g_1(y)$  where  $f_1(0)g_1(0) \neq 0$ . The elements  $f_1(x)$ ,  $g_1(y)$  are units in  $k[[x, y]]$  and can be written as  $\beta^{m_1}$ ,  $\gamma^{n_1}$  for some  $\beta, \gamma \in k[[x, y]]$ . Define  $X = x\beta$  and  $Y = y\gamma$ . Then  $A_1 \simeq k[[x, y]]/(g(y) - f(x)) \simeq k[[X, Y]]/(Y^{n_1} - X^{m_1})$ .) By Lemma 4, it follows that  $\dim(\bar{A}_i/A_i) = \{(m_i - 1)(n_i - 1) - 1 + d_i\}/2$ .

STEP 2. If  $|m - n| \leq 1$ , then the projective curve is non-singular except for those points belonging to  $\text{Sing}(\Gamma_0)$ . It is easy to check that  $(N - 1)(N - 2)/2 = \{(m - 1)(n - 1) + 1 - d\}/2$ , where  $N = \max\{m, n\}$ .

STEP 3. Consider the case  $m \geq n + 2$ . (The case  $m \leq n - 2$  is similar and will be omitted.)

Consider the homogenized polynomial equation  $X_0^m g(X_2/X_0) = X_0^n f(X_1/X_0)$  where  $x = X_1/X_0$ ,  $y = X_2/X_0$  and we shall write  $f(x) = \prod_{1 \leq i \leq m} (x + \lambda_i)$ ,  $g(y) = \prod_{1 \leq j \leq n} (y + \rho_j)$ . The only singular point of  $\Gamma$  other than those belonging to  $\text{Sing}(\Gamma_0)$  is  $P = (0 : 0 : 1)$ . Let  $z = X_0/X_2$ ,  $w = X_1/X_2$ . The dehomogenized polynomial becomes  $z^{m-n} \prod_{1 \leq j \leq n} (1 + \rho_j z) = \prod_{1 \leq i \leq m} (w + \lambda_i z)$ . It suffices to compute  $\delta_P = \dim(\bar{A}/A)$ , where

$$A = k[w, z]_{(w,z)} / (z^{m-n} \prod_{1 \leq j \leq n} (1 + \rho_j z) - \prod_{1 \leq i \leq m} (w + \lambda_i z)),$$

the local ring of  $\Gamma$  at the point  $P$ . Note that the multiplicity at the point  $P$  is  $m - n$ .

STEP 4. The element  $\prod_{1 \leq j \leq n} (1 + \rho_j z)$  is a unit in  $A$ . Call it  $\epsilon$ .

In the local ring  $A$ , consider the relation:  $\epsilon z^{m-n} = \prod_{1 \leq i \leq m} (w + \lambda_i z)$ . Define  $u = z/w$  in the quotient field of  $A$ . The above relation becomes  $\epsilon u^{m-n} = w^n \prod_{1 \leq i \leq m} (1 + \lambda_i u)$ .

Write  $\prod_{1 \leq i \leq m} (1 + \lambda_i u) = \sum_{0 \leq i \leq m} a_i u^i$ , where  $a_i \in k$  and  $a_0 = 1$ . Then we get  $\epsilon u^{m-n} - w^n \sum_{m-n \leq i \leq m} a_i u^i = w^n \sum_{0 \leq i \leq m-n-1} a_i u^i$ . Hence

$$u^{m-n}(\epsilon - \sum_{m-n \leq i \leq m} a_i w^n u^{i-m+n}) = \sum_{0 \leq i \leq m-n-1} a_i w^n u^i.$$

As

$$\epsilon - \sum_{m-n \leq i \leq m} a_i w^n u^{i-m+n} = \epsilon - a_{m-n} w^n - a_{m-n+1} w^{n-1} z - \cdots - a_m z^m$$

is a unit in  $A$ , we find

$$(1) \quad u^{m-n} + \alpha_1 u^{m-n-1} + \alpha_2 u^{m-n-2} + \cdots + \alpha_{m-n} = 0,$$

where  $\alpha_i \in w^n A$ . It follows that  $u \in \bar{A}$ .

Clearly,  $A \subset B \subset \bar{A}$ , where

$$B = A[u] = k[w, u]_{(w, u)} / (\epsilon u^{m-n} - w^n \prod_{1 \leq i \leq m} (1 + \lambda_i u)).$$

Thus  $\delta_P = \dim(\bar{A}/A) = \dim(B/A) + \dim(\bar{A}/B)$ .

STEP 5. We claim that  $\dim(B/A) = (m-n)(m-n-1)/2$ .

First we will show that

$$(2) \quad B = A + \sum_{0 \leq i < j \leq m-n-1} k \cdot w^i u^j.$$

Let  $C$  be the completion of  $A$ . Then  $C = k[[w, z]] / (\epsilon z^{m-n} - \prod_{1 \leq i \leq m} (w + \lambda_i z))$ .

Since  $B/A = A[u]/A$  is a finite-dimensional vector space over  $k$ , it is naturally isomorphic to  $(A[u]/A) \otimes_A C$ . Thus (2) is equivalent to

$$(3) \quad C[u] = C + \sum_{0 \leq i < j \leq m-n-1} k \cdot w^i u^j.$$

To check the validity of (3) it suffices to consider whether  $z^j u^j$  (where  $0 \leq j \leq m-n-1$ ) belongs to the right-hand-side of (3), because of Formula (1) and the relation  $uw = z$ . We will prove it by induction on  $j$ . If  $j = 0$ , it is trivial. Now assume  $j \geq 1$ . In case  $j \leq m-n-i-1$ , then  $z^j u^j = w^i u^{i+j}$  with  $i+j \leq m-n-1$  as required. If  $j \geq m-n-i$ , then  $z^j u^j = w^i u^{i+j}$  and  $i+j \geq m-n$ . Using (1) we find that  $u^{i+j}$  is a linear combination of  $w^n u^l$  with coefficients in  $A$ , where  $0 \leq l \leq m-n-1$ . It follows that, after modulo  $C$ ,  $z^j u^j$  is a linear combination of  $u^{l-n-i}$  (if  $l-n-i > 0$ ) where  $l \leq m-n-1$ . Note that  $l-n-i \leq m-2n-i-1 < j$ . Thus we have reduced  $z^j u^j$  to terms of lower exponents.

We shall show that the  $w^i u^j$  are linearly independent in  $B/A$  where  $0 \leq i < j \leq m-n-1$ . Suppose  $0 = \sum_{i < j} a_{ij} w^i u^j$  in  $B/A$  where  $a_{ij} \in k$  are not all zero. Hence  $\sum_{i < j} a_{ij} w^i u^j = \varphi \in A$ . Define  $p = \max\{j-i : a_{ij} \neq 0\}$ . Multiply the relation  $\sum_{i < j} a_{ij} w^i u^j = \varphi$  by  $w^p$ . We get  $\sum_{j-i=p} a_{ij} z^j = w\psi$  for some  $\psi \in A$ . Since  $1 \leq j \leq m-n-1$ , it follows that the multiplicity of the point  $P$  is  $\leq m-n-1$ , a contradiction.

STEP 6. Note that  $\bar{A}$  is also the normalization of  $B$ . We will compute  $\dim(\bar{A}/B)$ .

By [Se], p.59, Formula (3), the question can again be reduced to the complete case. Let  $D$  be the completion of  $B$ . Then

$$D = k[[w, u]]/(\epsilon u^{m-n} - w^n \prod_{1 \leq i \leq m} (1 + \lambda_i u)).$$

Find  $\alpha \in D$  such that  $\alpha^{m-n} = \epsilon^{-1} \prod_{1 \leq i \leq m} (1 + \lambda_i u)$ . Define  $U = u/\alpha$ . Then  $D \simeq k[[w, U]]/(U^{m-n} - w^n)$ . Now we can apply Lemma 4 to get  $\dim(\bar{D}/D) = \{(n-1)(m-n-1) - 1 + d\}/2$ .

STEP 7. Finally we find that  $\delta_p = \dim(\bar{A}/A) = \dim(B/A) + \dim(\bar{A}/B) = \{(m-1)(m-n-1) - 1 + d\}/2$ . Thus

$$(N-1)(N-2)/2 - \delta_p = \{(m-1)(n-1) + 1 - d\}/2$$

because  $N = \max\{m, n\} = m$ . This completes the proof of Theorem 2.

REMARKS.

(1) From the above proof, it is clear that Theorem 2 remains valid if  $\text{char } k = p > 0$  and  $p$  doesn't divide  $mn \prod_{1 \leq i \leq l} m_i n_i$ .

(2) Similarly, if  $p$  is a prime number and the affine curve is defined by  $y^p = \prod_{1 \leq i \leq l} (x - \lambda_i)^{m_i}$  such that the  $\lambda_i$  are distinct,  $1 \leq m_i < p$  and  $p$  doesn't divide  $\sum_{1 \leq i \leq l} m_i$ , then Theorem 2 (and its proof for this case) remains valid no matter what  $\text{char } k$  may be. Note that the latter assumption can always be achieved. For, if we denote  $\sum_{1 \leq i \leq l} m_i$  by  $m$  and suppose that  $m = pr$ , we may assume that  $\lambda_1 = 0$ . Divide both sides of the equation by  $x^m$ . Consider the new variables  $u = 1/x, v = y/x^r$ .

(3) On the other hand, if we assume that  $k$  is a perfect field (such that (i)  $p \nmid mn \prod_{1 \leq i \leq l} m_i n_i$  if  $\text{char } k = p > 0$ , or (ii)  $p$  is a prime number and the affine curve is defined by  $y^p = \prod_{1 \leq i \leq l} (x - \lambda_i)^{m_i}$  with ...) but not algebraically closed, then Theorem 2 is true because we can extend the constant field  $k$  to its algebraic closure at the beginning of the proof without affecting the genus by [Ch], p. 99.

REFERENCES

[BK] BRIESKORN, E. and H. KNÖRRER. *Plane Algebraic Curves*. Birkhäuser, Basel, 1986.  
 [Ca] CASAS-ALVERO, E. *Singularities of Plane Curves*. London Math. Soc. Lecture Note Series 276. Cambridge Univ. Press, Cambridge, 2000.

- [Ch] CHEVALLEY, C. *Introduction to the Theory of Algebraic Functions of One Variable*. Amer. Math. Soc., Providence, 1951.
- [DLS] DAVENPORT, H., D.J. LEWIS and A. SCHINZEL. Equations of the form  $f(x) = g(y)$ . *Quart. J. Math. Oxford Ser. (2)* 12 (1961), 304–312.
- [Fe] FEIT, W. Some consequences of the classification of finite simple groups. In: *Proc. Symp. Pure Math.* 37 (1980), 175–181.
- [Hi] HIRONAKA, H. On the arithmetic genera and the effective genera of algebraic curves. *Mem. Coll. Sci. Univ. Kyoto Ser. A. Math.* 30 (1957), 177–195.
- [Mi] MIRANDA, R. *Algebraic Curves and Riemann Surfaces*. Graduate Studies in Math. 5. Amer. Math. Soc., Providence, 1995.
- [NZ] NIVEN, I. and H. S. ZUCKERMAN. *An Introduction to the Theory of Numbers*, 2<sup>nd</sup> ed. Wiley, New York, 1966.
- [Pr] PRETZEL, O. *Codes and Algebraic Curves*. Clarendon Press, Oxford, 1998.
- [Se] SERRE, J.-P. *Algebraic Groups and Class Fields*. Springer GTM 117. Springer-Verlag, Berlin, 1988.

(Reçu le 13 novembre 2003)

Ming-Chang Kang

Department of Mathematics  
National Taiwan University  
Taipei, Taiwan  
*e-mail*: kang@math.ntu.edu.tw