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This proves that $\text{sp}(L)$ is finite. If $e^{\lambda_1 z}, \dots, e^{\lambda_s z}$ are the only exponential functions contained in L , then every exponential polynomial contained in L must be of the form $\sum_{i=1}^s p_i(z) e^{\lambda_i z}$, where p_1, \dots, p_s are polynomials. Since the set of all polynomials is dense in $C(\mathbf{R})$ and $L \neq C(\mathbf{R})$, it follows that the degrees of p_1, \dots, p_s must be bounded. As the set of exponential polynomials is dense in L , we find that each element of L is an exponential polynomial, which completes the proof of Theorem 2 in the case when $h \equiv 0$.

The general case can be reduced to the previous one in the same way as in the proof of Theorem 1. Again, it is enough to show that f_n is an exponential polynomial. Since $\Delta_b f_n$ satisfies the homogeneous version of (1), it follows that $\Delta_b f_n$ is an exponential polynomial for every b . Therefore, by Carroll's theorem [2], f_n is also an exponential polynomial. \square

REFERENCES

- [1] BAKER, J. A. Functional equations, distributions and approximate identities. *Canad. J. Math.* 42 (1990), 696–708.
- [2] CARROLL, F. W. A difference property for polynomials and exponential polynomials on Abelian locally compact groups. *Trans. Amer. Math. Soc.* 114 (1965), 147–155.
- [3] EVEREST, G. R. and A. J. VAN DEN POORTEN. Factorization in the ring of exponential polynomials. *Proc. Amer. Math. Soc.* 125 (1997), 1293–1298.
- [4] JÁRAI, A. On Lipschitz property of solutions of functional equations. *Aequationes Math.* 47 (1994), 69–78.
- [5] KAHANE, J.-P. Sur quelques problèmes d'unicité et de prolongement, relatifs aux fonctions approchables par des sommes d'exponentielles. *Ann. Inst. Fourier (Grenoble)* 5 (1953–54), 39–130.
- [6] ———. *Lectures on Mean Periodic Functions*. Tata Institute, 1956.
- [7] KELETI, T. Difference functions of periodic measurable functions. *Fund. Math.* 157 (1998), 15–32.
- [8] VAN DEN POORTEN, A. J. and R. TIJDEMAN. On common zeros of exponential polynomials. *L'Enseignement Math.* (2) 21 (1975), 57–67.
- [9] RITT, J. F. A factorization theory for functions $\sum_{i=1}^n a_i e^{\alpha_i x}$. *Trans. Amer. Math. Soc.* 29 (1927), 584–596.
- [10] SCHWARTZ, L. Théorie générale des fonctions moyenne-périodiques. *Ann. of Math.* (2) 48 (1947), 857–929.
- [11] SHAPIRO, H. S. The expansion of mean-periodic functions in series of exponentials. *Comm. Pure Appl. Math.* 11 (1958), 1–21.

- [12] ŚWIATAK, H. On the regularity of the locally integrable solutions of the functional equations $\sum_{i=1}^k a_i(x, t)f(x+\phi_i(t)) = b(x, t)$. *Aequationes Math.* 1 (1968), 6–19.

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