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This proves that $\text{sp}(L)$ is finite. If $e^{\lambda_1 z}, \dots, e^{\lambda_s z}$ are the only exponential functions contained in L , then every exponential polynomial contained in L must be of the form $\sum_{i=1}^s p_i(z) e^{\lambda_i z}$, where p_1, \dots, p_s are polynomials. Since the set of all polynomials is dense in $C(\mathbf{R})$ and $L \neq C(\mathbf{R})$, it follows that the degrees of p_1, \dots, p_s must be bounded. As the set of exponential polynomials is dense in L , we find that each element of L is an exponential polynomial, which completes the proof of Theorem 2 in the case when $h \equiv 0$.

The general case can be reduced to the previous one in the same way as in the proof of Theorem 1. Again, it is enough to show that f_n is an exponential polynomial. Since $\Delta_b f_n$ satisfies the homogeneous version of (1), it follows that $\Delta_b f_n$ is an exponential polynomial for every b . Therefore, by Carroll's theorem [2], f_n is also an exponential polynomial. \square

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