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respectively. Denote by  $\hat{t}_0$  the largest value of  $t$  for which the curve  $\Gamma$  has a simple spherical inflection in  $U_0$ . Denote by  $\hat{t}_1$  the smallest value of  $t$  for which the curve  $\Gamma$  has a simple spherical inflection in  $U_1$ . We proved that between  $\Gamma(\hat{t}_0)$  and  $\Gamma(\hat{t}_1)$  there is at least one vertex of  $\Gamma$  of odd order. We can join  $\Gamma$  to  $\gamma$  by a homotopy  $\gamma_s$  such that at each  $s \in [0, 1]$  the curve  $\gamma_s$  has only simple spherical inflections. Thus each  $\gamma_s$  will have at least one vertex of odd order. This implies that  $\gamma$  will also have at least one vertex of odd order. This proves Proposition 4.  $\square$

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