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IDEAL SOLUTIONS OF THE TARRY-ESCOTT PROBLEM
OF DEGREES FOUR AND FIVE
AND RELATED DIOPHANTINE SYSTEMS

by Ajai CHOUDHRY

ABSTRACT. In this paper, we obtain parametric ideal non-symmetric solutions in integers of the Tarry-Escott problem of degrees four and five, that is, of the system of simultaneous equations $\sum_{i=1}^{k+1} a_i^r = \sum_{i=1}^{k+1} b_i^r$, $r = 1, 2, \dots, k$ where k is 4 or 5. We use these non-symmetric solutions to obtain parametric solutions of the two diophantine systems $\sum_{i=1}^{k+1} a_i^r = \sum_{i=1}^{k+1} b_i^r$, $r = 1, 2, \dots, k, k + 2$ where k is 4 or 5.

1. INTRODUCTION

This paper is a sequel to my earlier paper [1] regarding the Tarry-Escott problem. It would be recalled that very little is known about ideal non-symmetric solutions of the Tarry-Escott problem of degree k when $k > 3$. When $k = 4$, the only known parametric ideal non-symmetric solution of the Tarry-Escott problem is given in [1]. This solution is in terms of polynomials of degree 8 in two parameters. When $k = 5$, only a single numerical solution seems to have been published [2, p.27]. No non-symmetric solutions have been published for $k > 5$.

In this paper, we will obtain parametric ideal non-symmetric solutions of the Tarry-Escott problem of degrees four and five. The parametric solutions of the Tarry-Escott problem of degree four obtained in this paper are more general and much simpler as compared to the parametric solution of this problem given in [1].

It has already been shown in [1] how ideal non-symmetric solutions of the Tarry-Escott problem of degree k may be used to generate solutions of the system of equations

$$(1.1) \quad \sum_{i=1}^{k+1} a_i^r = \sum_{i=1}^{k+1} b_i^r, \quad r = 1, 2, \dots, k, k+2$$

by applying a theorem of Gloden [2, p.24]. Applying this procedure to the non-symmetric ideal solutions of degrees four and five obtained in this paper, we get parametric solutions of (1.1) when $k = 4$ or $k = 5$.

2. IDEAL NON-SYMMETRIC SOLUTIONS OF THE TARRY-ESCOTT PROBLEM OF DEGREE FOUR

To obtain ideal non-symmetric solutions of the Tarry-Escott problem of degree four, we have to obtain a solution of the system of equations

$$(2.1) \quad \sum_{i=1}^5 a_i^r = \sum_{i=1}^5 b_i^r, \quad r = 1, 2, 3, 4.$$

We first observe that the system of equations

$$(2.2) \quad X_1^r + X_2^r + X_3^r = Y_1^r + Y_2^r + Y_3^r, \quad r = 1, 2, 4,$$

reduces to

$$(2.3) \quad X_1^2 + X_1X_2 + X_2^2 = Y_1^2 + Y_1Y_2 + Y_2^2,$$

if we take $X_3 = -X_1 - X_2$ and $Y_3 = -Y_1 - Y_2$. A solution of (2.3) in terms of arbitrary parameters m, n, x, y , is given by

$$(2.4) \quad \begin{aligned} X_1 &= (m + 2n)x + (-m + n)y, \\ X_2 &= (-2m - n)x + (-m - 2n)y, \\ Y_1 &= (m - n)x + (-m - 2n)y, \\ Y_2 &= (-2m - n)x + (-m + n)y, \end{aligned}$$

and we now get

$$(2.5) \quad \begin{aligned} X_3 &= (m - n)x + (2m + n)y, \\ Y_3 &= (m + 2n)x + (2m + n)y. \end{aligned}$$

It follows from this solution of the system of equations (2.2) that if we take