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Autor: Choudhry, Ajai

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IDEAL SOLUTIONS OF THE TARRY-ESCOTT PROBLEM OF DEGREES FOUR AND FIVE AND RELATED DIOPHANTINE SYSTEMS

by Ajai CHOUDHRY

ABSTRACT. In this paper, we obtain parametric ideal non-symmetric solutions in integers of the Tarry-Escott problem of degrees four and five, that is, of the system of simultaneous equations $\sum_{i=1}^{k+1} a_i^r = \sum_{i=1}^{k+1} b_i^r$, $r = 1, 2, \ldots, k$ where k is 4 or 5. We use these non-symmetric solutions to obtain parametric solutions of the two diophantine systems $\sum_{i=1}^{k+1} a_i^r = \sum_{i=1}^{k+1} b_i^r$, $r = 1, 2, \ldots, k, k+2$ where k is 4 or 5.

1. INTRODUCTION

This paper is a sequel to my earlier paper [1] regarding the Tarry-Escott problem. It would be recalled that very little is known about ideal non-symmetric solutions of the Tarry-Escott problem of degree k when $k > 3$. When $k = 4$, the only known parametric ideal non-symmetric solution of the Tarry-Escott problem is given in [1]. This solution is in terms of polynomials of degree 8 in two parameters. When $k = 5$, only a single numerical solution seems to have been published [2, p. 27]. No non-symmetric solutions have been published for $k > 5$.

In this paper, we will obtain parametric ideal non-symmetric solutions of the Tarry-Escott problem of degrees four and five. The parametric solutions of the Tarry-Escott problem of degree four obtained in this paper are more general and much simpler as compared to the parametric solution of this problem given in [1].

It has already been shown in [1] how ideal non-symmetric solutions of the Tarry-Escott problem of degree k may be used to generate solutions of the system of equations

$$\sum_{i=1}^{k+1} a_i^r = \sum_{i=1}^{k+1} b_i^r\,, \quad r = 1, 2, \ldots, k, k+2 \tag{1.1}$$

by applying a theorem of Gloden [2, p. 24]. Applying this procedure to the non-symmetric ideal solutions of degrees four and five obtained in this paper, we get parametric solutions of (1.1) when $k = 4$ or $k = 5$.

2. Ideal non-symmetric solutions of the Tarry-Escott problem of degree four

To obtain ideal non-symmetric solutions of the Tarry-Escott problem of degree four, we have to obtain a solution of the system of equations

$$\sum_{i=1}^{5} a_i^r = \sum_{i=1}^{5} b_i^r\,, \qquad r = 1, 2, 3, 4. \tag{2.1}$$

We first observe that the system of equations

$$X_1^r + X_2^r + X_3^r = Y_1^r + Y_2^3 + Y_3^r\,, \qquad r = 1, 2, 4, \tag{2.2}$$

reduces to

$$X_1^2 + X_1X_2 + X_2^2 = Y_1^2 + Y_1Y_2 + Y_2^2\,, \tag{2.3}$$

if we take $X_3 = -X_1 - X_2$ and $Y_3 = -Y_1 - Y_2$. A solution of (2.3) in terms of arbitrary parameters m, n, x, y, is given by

$$\begin{aligned} X_1 &= (m+2n)x + (-m+n)y\,, \\ X_2 &= (-2m-n)x + (-m-2n)y\,, \\ Y_1 &= (m-n)x + (-m-2n)y\,, \\ Y_2 &= (-2m-n)x + (-m+n)y\,, \end{aligned} \tag{2.4}$$

and we now get

$$\begin{aligned} X_3 &= (m-n)x + (2m+n)y\,, \\ Y_3 &= (m+2n)x + (2m+n)y\,. \end{aligned} \tag{2.5}$$

It follows from this solution of the system of equations (2.2) that if we take