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induces an isomorphism

$$K_0(B\{e\}) \xrightarrow{\cong} K_0(BA_G).$$

Our strategy is as follows. We show that the Atiyah L^2 -Index Theorem holds in the special case of acyclic groups, and finish the proof combining the above embedding of a group into an acyclic group.

Proof of Theorem 2.1. If a group A is acyclic, the equation $\text{Index}_A = \text{Index}$ follows from the diagram

$$\begin{array}{ccccc} K_0(BA) & \xrightarrow{\text{Index}_A} & \mathbf{R} & \xleftarrow{\text{Index}} & K_0(BA) \\ \cong \uparrow & & \uparrow & & \cong \uparrow \\ K_0(B\{e\}) & \xrightarrow[\cong]{\text{Index}_{\{e\}}} & \mathbf{Z} & \xleftarrow[\cong]{\text{Index}} & K_0(B\{e\}) \end{array}$$

because $\text{Index}_{\{e\}} = \text{Index}$ on the bottom line. For a general group G , consider an embedding into an acyclic group A_G and complete the proof by using Lemma 3.1, together with Lemmas 4.1 and 4.2.

REFERENCES

- [1] ATIYAH, M.F. Elliptic operators, discrete groups and von Neumann algebras. *Astérisque* 32-3 (1976), 43-72.
- [2] ATIYAH, M.F. and I.M. SINGER. The index of elliptic operators III. *Ann. of Math.* (2) 87 (1968), 546-604.
- [3] BAUM, P. and A. CONNES. K -theory for Lie groups and foliations. *L'Enseignement Math.* (2) 46 (2000), 3-42.
- [4] BAUM, P. and R. DOUGLAS. K -homology and index theory. *Proceedings of Symposia in Pure Mathematics* 38, Part 1 (1982), 117-173.
- [5] BERRICK, A.J. and K. VARADARAJAN. Binate towers of groups. *Arch. Math.* 62 (1994), 97-111.
- [6] BERRICK, A.J., I. CHATTERJI and G. MISLIN. From acyclic groups to the Bass Conjecture for amenable groups. (Submitted for publication 2002.)
- [7] BERRICK, A.J. The acyclic group dichotomy. (Preprint in preparation.)
- [8] DICKS, W. and T. SCHICK. The spectral measure of certain elements of the complex group ring of a wreath product. *Geom. Dedicata* 93 (2002), 121-137.
- [9] ECKMANN, B. Introduction to l_2 -methods in topology: reduced l_2 -homology, harmonic chains, l_2 -Betti numbers. (Notes prepared by Guido Mislin.) *Israel J. Math.* 117 (2000), 183-219.
- [10] HIGSON, N. and J. ROE. *Analytic K -Homology*. Oxford Mathematical Monographs, Oxford University Press, 2000.

- [11] KAN, D.M. and W.P. THURSTON. Every connected space has the homology of a $K(\pi, 1)$. *Topology* 15 (1976), 253–258.
- [12] KASPAROV, G. K -theory, group C^* -algebras, and higher signatures (Conspetus). Novikov conjectures, index theorems and rigidity, Vol. 1 (Oberwolfach, 1993), 101–146. London Math. Soc. Lecture Note Ser. 226. Cambridge Univ. Press, 1995.
- [13] SOLOVYOV, Y.P. and E.V. TROITSKY. C^* -Algebras and Elliptic Operators in Differential Topology. (Translated from the 1996 Russian original by Troitsky.) Translations of Mathematical Monographs, 192. Amer. Math.Soc., Providence (R.I.), 2001.
- [14] VALETTE, A. *Introduction to the Baum-Connes Conjecture*. (Notes taken by Indira Chatterji. With an appendix by Guido Mislin.) Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2002.

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