

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 49 (2003)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ATIYAH'S L^2 -INDEX THEOREM

Autor: Chatterji, Indira / Mislin, Guido

Bibliographie

DOI: <https://doi.org/10.5169/seals-66679>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 15.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

induces an isomorphism

$$K_0(B\{e\}) \xrightarrow{\cong} K_0(BA_G).$$

Our strategy is as follows. We show that the Atiyah L^2 -Index Theorem holds in the special case of acyclic groups, and finish the proof combining the above embedding of a group into an acyclic group.

Proof of Theorem 2.1. If a group A is acyclic, the equation $\text{Index}_A = \text{Index}$ follows from the diagram

$$\begin{array}{ccccc} K_0(BA) & \xrightarrow{\text{Index}_A} & \mathbf{R} & \xleftarrow{\text{Index}} & K_0(BA) \\ \cong \uparrow & & \uparrow & & \cong \uparrow \\ K_0(B\{e\}) & \xrightarrow[\cong]{\text{Index}_{\{e\}}} & \mathbf{Z} & \xleftarrow[\cong]{\text{Index}} & K_0(B\{e\}) \end{array}$$

because $\text{Index}_{\{e\}} = \text{Index}$ on the bottom line. For a general group G , consider an embedding into an acyclic group A_G and complete the proof by using Lemma 3.1, together with Lemmas 4.1 and 4.2.

REFERENCES

- [1] ATIYAH, M. F. Elliptic operators, discrete groups and von Neumann algebras. *Astérisque* 32–3 (1976), 43–72.
- [2] ATIYAH, M. F. and I. M. SINGER. The index of elliptic operators III. *Ann. of Math.* (2) 87 (1968), 546–604.
- [3] BAUM, P. and A. CONNES. K -theory for Lie groups and foliations. *L'Enseignement Math.* (2) 46 (2000), 3–42.
- [4] BAUM, P. and R. DOUGLAS. K -homology and index theory. *Proceedings of Symposia in Pure Mathematics* 38, Part 1 (1982), 117–173.
- [5] BERRICK, A. J. and K. VARADARAJAN. Binate towers of groups. *Arch. Math.* 62 (1994), 97–111.
- [6] BERRICK, A. J., I. CHATTERJI and G. MISLIN. From acyclic groups to the Bass Conjecture for amenable groups. (Submitted for publication 2002.)
- [7] BERRICK, A. J. The acyclic group dichotomy. (Preprint in preparation.)
- [8] DICKS, W. and T. SCHICK. The spectral measure of certain elements of the complex group ring of a wreath product. *Geom. Dedicata* 93 (2002), 121–137.
- [9] ECKMANN, B. Introduction to l_2 -methods in topology: reduced l_2 -homology, harmonic chains, l_2 -Betti numbers. (Notes prepared by Guido Mislin.) *Israel J. Math.* 117 (2000), 183–219.
- [10] HIGSON, N. and J. ROE. *Analytic K-Homology*. Oxford Mathematical Monographs, Oxford University Press, 2000.

- [11] KAN, D. M. and W. P. THURSTON. Every connected space has the homology of a $K(\pi, 1)$. *Topology* 15 (1976), 253–258.
- [12] KASPAROV, G. K -theory, group C^* -algebras, and higher signatures (Conspic-tus). Novikov conjectures, index theorems and rigidity, Vol. 1 (Ober-wolfach, 1993), 101–146. London Math. Soc. Lecture Note Ser. 226. Cambridge Univ. Press, 1995.
- [13] SOLOVYOV, Y. P. and E. V. TROITSKY. *C^* -Algebras and Elliptic Operators in Differential Topology*. (Translated from the 1996 Russian original by Troitsky.) Translations of Mathematical Monographs, 192. Amer. Math.Soc., Providence (R.I.), 2001.
- [14] VALETTE, A. *Introduction to the Baum-Connes Conjecture*. (Notes taken by Indira Chatterji. With an appendix by Guido Mislin.) Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2002.

(Reçu le 15 septembre 2002)

Indira Chatterji

Mathematics Department
Cornell University
Ithaca NY 14853
U. S. A.
e-mail : indira@math.cornell.edu

Guido Mislin

Mathematics Department
ETHZ
8092 Zürich
Switzerland
e-mail : mislin@math.ethz.ch

Vide-leer-empty