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induces an isomorphism

$$K_0(B\{e\}) \xrightarrow{\cong} K_0(BA_G).$$

Our strategy is as follows. We show that the Atiyah L^2-Index Theorem holds in the special case of acyclic groups, and finish the proof combining the above embedding of a group into an acyclic group.

Proof of Theorem 2.1. If a group A is acyclic, the equation $\mathrm{Index}_A = \mathrm{Index}$ follows from the diagram

$$\begin{tikzcd}
K_0(BA) \arrow[r, "\mathrm{Index}_A"] & \mathbf{R} & K_0(BA) \arrow[l, "\mathrm{Index}"'] \\
K_0(B\{e\}) \arrow[u, "\cong"] \arrow[r, "\mathrm{Index}_{\{e\}}", "\cong"'] & \mathbf{Z} \arrow[u] & K_0(B\{e\}) \arrow[u, "\cong"] \arrow[l, "\mathrm{Index}"', "\cong"]
\end{tikzcd}$$

because $\mathrm{Index}_{\{e\}} = \mathrm{Index}$ on the bottom line. For a general group G, consider an embedding into an acyclic group A_G and complete the proof by using Lemma 3.1, together with Lemmas 4.1 and 4.2.

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