

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 49 (2003)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** LECTURES ON QUASI-INVARIANTS OF COXETER GROUPS AND THE CHEREDNIK ALGEBRA  
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**Kapitel:** 1.6 The ring of differential operators on  $X_m$   
**DOI:** <https://doi.org/10.5169/seals-66677>

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From this we deduce

THEOREM 1.14 ([EG2, BEG, FeV], conjectured in [FV]). *The ring  $Q_m$  of  $m$ -quasi-invariants is Gorenstein.*

*Proof.* By Stanley's theorem (see [Eis]), a positively graded Cohen-Macaulay domain  $A$  is Gorenstein iff its Poincaré series is a rational function  $h(t)$  satisfying the equation  $h(t^{-1}) = (-1)^n t^l h(t)$ , where  $l$  is an integer and  $n$  is the dimension of the spectrum of  $A$ . Thus the result follows immediately from Proposition 1.13.  $\square$

## 1.6 THE RING OF DIFFERENTIAL OPERATORS ON $X_m$

Finally, let us introduce the ring  $\mathcal{D}(X_m)$  of differential operators on  $X_m$ , that is the ring of differential operators with coefficients in  $\mathbf{C}(\mathfrak{h})$  mapping  $Q_m$  to  $Q_m$ . It is clear that this definition coincides with Grothendieck's well-known definition ([Bj]).

THEOREM 1.15 ([BEG]).  *$\mathcal{D}(X_m)$  is a simple algebra.*

REMARK 1.16. a) The ring of differential operators on a smooth affine algebraic variety is always simple (see [Bj], Chapter 3).

b) By a result of M. van den Bergh [VdB], for a non-smooth variety, the simplicity of the ring of differential operators implies the Cohen-Macaulay property of this variety.

## 2. LECTURE 2

We will now see how the ring  $Q_m$  appears in the theory of completely integrable systems.

### 2.1 HAMILTONIAN MECHANICS AND INTEGRABLE SYSTEMS

Recall the basic setup of Hamiltonian mechanics [Ar]. Consider a mechanical system with configuration space  $X$  (a smooth manifold). Then the phase space of this system is  $T^*X$ , the cotangent bundle on  $X$ . The space  $T^*X$  is naturally a symplectic manifold, and in particular we have an operation of Poisson bracket on functions on  $T^*X$ . A point of  $T^*X$  is a pair  $(x, p)$ , where  $x \in X$  is the position and  $p \in T_x^*X$  is the momentum. Such pairs are