

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 49 (2003)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP
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Anhang: Appendix A. Proof of Lemma 4.4
DOI: <https://doi.org/10.5169/seals-66691>

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REMARK 6.2. Z. Shahbazi has proved that if \mathcal{G} is a gerbe with connection over a manifold M , with curvature 3-form η , and $\Phi: N \rightarrow M$ is a map with $\Phi^*\eta + d\omega = 0$, then the pull-back gerbe $\Phi^*\mathcal{G}$ admits a pseudo-line bundle, with ω as its error 2-form, if and only if the pair (η, ω) defines an integral element of the relative de Rham cohomology $H^3(\Phi, \mathbf{R})$. This means that for any smooth 2-cycle $S \subset N$, and any smooth 3-chain $B \subset M$ with boundary $\Phi(S)$, one must have $\int_B \eta - \int_S \omega \in \mathbf{Z}$. The particular case where the target of Φ is a Lie group G is relevant for the pre-quantization of group-valued moment maps [1].

APPENDIX A. PROOF OF LEMMA 4.4

In this Appendix we prove Lemma 4.4, concerning the construction of a certain cover U_I of M from a given cover V_j . Write $M = \coprod_I A_I$ where

$$A_I = \bigcap_{i \in I} V_i \setminus \bigcup_{j \notin I} V_j.$$

Notice that $\bar{A}_I \subset \bigcup_{J \subset I} A_J$. By induction on the cardinality $k = |I|$ we will construct open sets $U_I \subset V_I$, having the following properties:

- (a) the closure \bar{U}_I does not meet \bar{U}_J for $|J| \leq |I|$ unless $J \subset I$,
- (b) each \bar{A}_I is contained in the union of U_J with $J \subset I$.

The induction starts at $k = 0$, taking $U_\emptyset = \emptyset$. Suppose we have constructed open sets U_I with $\bar{U}_I \subset V_I$ for $|I| < k$, such that the properties (a), (b) hold for all $|I| < k$. For $|I| = k$ consider the subsets

$$B_I := A_I \setminus \left(\bigcup_{J \subset I, |J| < k} U_J \right).$$

Note that (unlike A_I) the set B_I is closed. B_I does not meet \bar{A}_J unless $I \subset J$, and it also does not meet \bar{U}_J for $|J| < k$ unless $J \subset I$. That is, B_I is disjoint from

$$C_I := \bigcup_{J \not\subset I, |J| < k} \bar{U}_J \cup \bigcup_{K \not\subset I} \bar{A}_K.$$

Choose open sets U_I for $|I| = k$ with $B_I \subset U_I \subset \bar{U}_I \subset M \setminus C_I$, and such that the closures of the sets U_I for distinct I with $|I| = k$ are disjoint. The new collection of subsets will satisfy the properties (a), (b) for $|I| \leq k$. We next show that $V'_i = M \setminus \bigcup_{J \not\supset i} \bar{U}_J$ is a cover of M . Write $M = \coprod_I D_I$ with $D_I = \bar{U}_I \setminus \bigcup_{|J| < |I|} \bar{U}_J$. Then $D_I \cap \bar{U}_J = \emptyset$ unless $I \subset J$, so D_I is contained

in each V'_i with $i \in I$. In particular $\bigcup_i V'_i = M$. Finally $\overline{V'_i} \subset \bigcup_{I \ni i} \overline{U}_I \subset V_i$. This completes the proof of Lemma 4.4. Note that if the V_i were invariant under an action of a compact group G , the U_I could be taken G -invariant also.

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