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5. THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP

In this section we explain our construction of the basic gerbe over a compact, simple, simply connected Lie group.

5.1 NOTATION

Let G be a compact, simple, simply connected Lie group, with Lie algebra \mathfrak{g} . For any action of $G \times M \rightarrow M$, $(g, m) \mapsto g.m$ on a manifold M , we will denote by G_m the stabilizer group of a point $m \in M$. If $M = G$ or $M = \mathfrak{g}$, we will always consider the adjoint action of G unless specified otherwise. For instance, G_g for denotes the centralizer of an element $g \in G$.

Choose a maximal torus T of G , with Lie algebra \mathfrak{t} . Let $\Lambda = \ker(\exp|_{\mathfrak{t}})$ be the integral lattice and $\Lambda^* \subset \mathfrak{t}^*$ its dual, the (real) weight lattice. Equivalently, Λ is characterized as the lattice generated by the coroots $\check{\alpha}$ for the (real) roots α . Recall that the *basic inner product* \cdot on \mathfrak{g} is the unique invariant inner product such that $\check{\alpha} \cdot \check{\alpha} = 2$ for all long roots α . Throughout this paper, we will use the basic inner product to identify $\mathfrak{g}^* \cong \mathfrak{g}$. Choose a collection of simple roots $\alpha_1, \dots, \alpha_d \in \Lambda^*$ and let $\mathfrak{t}_+ = \{\xi \mid \alpha_j \cdot \xi \geq 0, j = 1, \dots, d\}$ be the corresponding positive Weyl chamber. The fundamental alcove \mathfrak{A} is the subset cut out from \mathfrak{t}_+ by the additional inequality $\alpha_0 \cdot \xi \geq -1$ where α_0 is the lowest root.

The fundamental alcove parametrizes conjugacy classes in G , in the sense that each conjugacy class contains a unique point $\exp \xi$ with $\xi \in \mathfrak{A}$. The quotient map will be denoted $q: G \rightarrow \mathfrak{A}$. Let μ_0, \dots, μ_d be the vertices of \mathfrak{A} , with $\mu_0 = 0$. For any $I \subseteq \{0, \dots, d\}$, all group elements $\exp \xi$ with ξ in the open face spanned by μ_j with $j \in I$ have the same centralizer, denoted G_I . In particular, G_j will denote the centralizer of $\exp \mu_j$.

For each j let $\mathfrak{A}_j \subset \mathfrak{A}$ be the open star at μ_j , i.e. the union of all open faces containing μ_j in their closure. Put differently, \mathfrak{A}_j is the complement of the closed face opposite to the vertex μ_j . We will work with the open cover of G given by the pre-images, $V_j = q^{-1}(\mathfrak{A}_j)$. More generally let $\mathfrak{A}_I = \bigcap_{j \in I} \mathfrak{A}_j$, and $V_I := q^{-1}(\mathfrak{A}_I)$. The flow-out $S_I = G_I \cdot \exp(\mathfrak{A}_I)$ of $\exp(\mathfrak{A}_I) \subset T$ under the action of G_I is an open subset of G_I , and is a slice for the conjugation action of G . That is,

$$G \times_{G_I} S_I = V_I.$$

We let $\pi_I: V_I \rightarrow G/G_I$ denote the projection to the base.