

## 13.3 Proof of Theorem 13.2

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Thus  $a_I \omega_1 b_I b_I^{-1} \omega_1^{-1} \omega^{-1} a_I^{-1} \in \text{Im}(\alpha)$ ,  $a'_I \omega_1 b'_I b'_I^{-1} \omega_1^{-1} \omega^{-1} a_I'^{-1} \in \text{Im}(\alpha)$ ,  $a_I \omega_1 b'_I b_I^{-1} \omega_1^{-1} \omega^{-1} a_I'^{-1} \notin \text{Im}(\alpha)$ . Now  $(a_I \omega^{-1} a_I'^{-1})^{-1} a_I \omega^{-1} a_I^{-1} (a_I \omega^{-1} a_I'^{-1}) = a_I' \omega^{-1} a_I'^{-1} \in \text{Im}(\alpha)$ , whereas  $a_I \omega^{-1} a_I'^{-1} \notin \text{Im}(\alpha)$  and  $a_I \omega^{-1} a_I^{-1} \in \text{Im}(\alpha)$ . We thus get a contradiction to the malnormality of  $\text{Im}(\alpha)$  in  $F_n$ . This completes the proof.  $\square$

### 13.3 PROOF OF THEOREM 13.2

From Lemmas 13.6 and 13.7, the Cayley complex  $\mathcal{C}(G_\alpha)$  is the mapping-telescope of a strongly hyperbolic forest-map, equipped with the standard metric. A Cayley complex is connected. Thus, from Theorem 12.4,  $\mathcal{C}(G_\alpha)$  is a Gromov-hyperbolic metric space for any mapping-telescope standard metric. From Lemma 13.5 the group  $G_\alpha$  acts cocompactly, properly discontinuously and isometrically on  $\mathcal{C}(G_\alpha)$  equipped with a mapping-telescope standard metric. A classical lemma of geometric group theory (usually attributed to Effremovich, Svàrc, Milnor – see [19] or [17] for instance), applied to quasi geodesic metric spaces, tells us that  $G_\alpha$  and  $\mathcal{C}(G_\alpha)$  are quasi-isometric so that  $G_\alpha$  is a hyperbolic group.  $\square$

REMARK 13.8. Another way of stating our main theorem about ‘forest-stacks’, using the language of trees of spaces, goes roughly as follows: “An oriented  $\mathbf{R}$ -tree of  $\mathbf{R}$ -trees with the gluing-maps satisfying the conditions of hyperbolicity and strong hyperbolicity with uniform constants is Gromov-hyperbolic.” Here ‘oriented  $\mathbf{R}$ -tree’ means an  $\mathbf{R}$ -tree  $T$  equipped with an orientation going from the domain to the image of each attaching-map, and a surjective continuous map  $f: T \rightarrow \mathbf{R}$  respecting this orientation. As a corollary of our theorem, and in order to illustrate it, we chose to concentrate on mapping-telescopes. We could as well consider spaces similar to mapping-telescopes but where we allow the attaching-maps not to be the same at each step. Our only requirement is to have uniform constants of quasi-isometry, hyperbolicity and so on. Also, with respect to groups, a corollary could have been stated dealing with HNN-extensions rather than just semi-direct products.

Another result which easily follows from our work could be more or less stated as follows. “Let  $T$  be a tree of spaces  $X_i$ ,  $i = 0, 1, \dots$ . Let  $\psi: T \rightarrow T$  be a map of  $T$  such that the mapping-telescope of each  $X_i$  under  $\psi$  is Gromov-hyperbolic. If  $\psi$  induces a hyperbolic map on the tree resulting of the collapsing of each  $X_i$  to a point, then the mapping-telescope of the tree of spaces  $T$  under  $\psi$  is Gromov-hyperbolic.” We leave the precise statement of such corollaries to the reader. Together with [14] where a new proof of the

Bestvina-Feighn theorem is given for mapping-tori of surface groups, the last one gives, thanks to [26], a new proof of the full version of the Combination Theorem for mapping-tori of hyperbolic groups, namely: “If  $G$  is a hyperbolic group and  $\alpha$  is a hyperbolic automorphism of  $G$ , then  $G \rtimes_{\alpha} \mathbf{Z}$  is a hyperbolic group.”

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