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12.2 PROOF OF THEOREM 12.4

LEMMA 12.5. *Let (Γ, d_Γ) be an \mathbf{R} -forest. Let ψ be a weakly bi-Lipschitz forest-map of (Γ, d_Γ) . Let (K_ψ, f, σ_t) be the mapping-telescope of (ψ, Γ) , equipped with a structure of forest-stack as defined in Section 2. Then the semi-flow $(\sigma_t)_{t \in \mathbf{R}^+}$ is a bounded-cancellation and bounded-dilatation semi-flow with respect to any horizontal d_Γ -metric (see Lemma 12.1).*

Proof. The horizontal metric \mathcal{H} agrees with the metric d_Γ on all the strata $f^{-1}(n)$, $n \in \mathbf{Z}$ (see Lemma 12.1). Consider any horizontal geodesic g in the stratum $f^{-1}(0)$. If ψ is weakly bi-Lipschitz with constants μ_0 and K_0 , then for any integer $n \geq 0$, we have $|[g]_n|_n \geq \frac{1}{\mu_0^n} |g|_0 - K_0 \left(\frac{1}{\mu_0^{n-1}} + \frac{1}{\mu_0^{n-2}} + \dots + 1 \right)$. Since $0 < \frac{1}{\mu_0} < 1$, the sum tends to $\frac{\mu_0}{\mu_0 - 1}$ as $n \rightarrow +\infty$. Setting $\lambda_- = \frac{1}{\mu_0}$ and $K = K_0 \frac{\mu_0}{\mu_0 - 1}$, this proves the inequality of item (1) for horizontal geodesics in $f^{-1}(n)$, $n \in \mathbf{Z}$, and an integer time t . For the case in which t is any positive real number and $g \in f^{-1}(r)$, r any real number, just decompose $\sigma_t = \sigma_{t-E[t]} \circ \sigma_{E[t-(E[r]+1-r)]} \circ \sigma_{E[r]+1-r}$. The map σ_t is a homeomorphism from $f^{-1}(r)$ onto $f^{-1}(r+t)$ for any $t \in [0, E[r]+1-r)$. That is, for any real r , $|[g]_{r+t}|_{r+t} = |\sigma_t(g)|_{r+t}$ for $t \in [0, E[r]+1-r)$. The monotonicity of the maps $l_{r,g}$ (see Lemma 12.1, item (2)) implies, for any r and $t \in [0, E[r]+1-r)$, that $|\sigma_t(g)|_{r+t} \geq \frac{1}{\mu_0} |g|_r$. The conclusion follows. \square

LEMMA 12.6. *With the assumptions and notation of Lemma 12.5, if the map ψ is a (strongly) hyperbolic forest-map of (Γ, d_Γ) then the semi-flow $(\sigma_t)_{t \in \mathbf{R}^+}$ is (strongly) hyperbolic with respect to any horizontal d_Γ -metric.*

The proof is similar to that of Lemma 12.5. \square

Proof of Theorem 12.4. By Lemmas 12.5 and 12.6, a mapping-telescope admits a structure of forest-stack $(\tilde{X}, f, \sigma_t, \mathcal{H})$ with horizontal metric \mathcal{H} such that the semi-flow $(\sigma_t)_{t \in \mathbf{R}^+}$ is a strongly hyperbolic semi-flow with respect to \mathcal{H} . Hence Theorem 4.4 implies Theorem 12.4. \square

13. ABOUT MAPPING-TORUS GROUPS

We first recall the definition of a *hyperbolic endomorphism* of a group introduced by Gromov [19].