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## 12.2 PROOF OF THEOREM 12.4

LEMMA 12.5. *Let  $(\Gamma, d_\Gamma)$  be an  $\mathbf{R}$ -forest. Let  $\psi$  be a weakly bi-Lipschitz forest-map of  $(\Gamma, d_\Gamma)$ . Let  $(K_\psi, f, \sigma_t)$  be the mapping-telescope of  $(\psi, \Gamma)$ , equipped with a structure of forest-stack as defined in Section 2. Then the semi-flow  $(\sigma_t)_{t \in \mathbf{R}^+}$  is a bounded-cancellation and bounded-dilatation semi-flow with respect to any horizontal  $d_\Gamma$ -metric (see Lemma 12.1).*

*Proof.* The horizontal metric  $\mathcal{H}$  agrees with the metric  $d_\Gamma$  on all the strata  $f^{-1}(n)$ ,  $n \in \mathbf{Z}$  (see Lemma 12.1). Consider any horizontal geodesic  $g$  in the stratum  $f^{-1}(0)$ . If  $\psi$  is weakly bi-Lipschitz with constants  $\mu_0$  and  $K_0$ , then for any integer  $n \geq 0$ , we have  $|[g]_n|_n \geq \frac{1}{\mu_0^n} |g|_0 - K_0(\frac{1}{\mu_0^{n-1}} + \frac{1}{\mu_0^{n-2}} + \dots + 1)$ . Since  $0 < \frac{1}{\mu_0} < 1$ , the sum tends to  $\frac{\mu_0}{\mu_0 - 1}$  as  $n \rightarrow +\infty$ . Setting  $\lambda_- = \frac{1}{\mu_0}$  and  $K = K_0 \frac{\mu_0}{\mu_0 - 1}$ , this proves the inequality of item (1) for horizontal geodesics in  $f^{-1}(n)$ ,  $n \in \mathbf{Z}$ , and an integer time  $t$ . For the case in which  $t$  is any positive real number and  $g \in f^{-1}(r)$ ,  $r$  any real number, just decompose  $\sigma_t = \sigma_{t-E[t]} \circ \sigma_{E[t-(E[r]+1-r)]} \circ \sigma_{E[r]+1-r}$ . The map  $\sigma_t$  is a homeomorphism from  $f^{-1}(r)$  onto  $f^{-1}(r+t)$  for any  $t \in [0, E[r]+1-r]$ . That is, for any real  $r$ ,  $|[g]_{r+t}|_{r+t} = |\sigma_t(g)|_{r+t}$  for  $t \in [0, E[r]+1-r]$ . The monotonicity of the maps  $l_{r,g}$  (see Lemma 12.1, item (2)) implies, for any  $r$  and  $t \in [0, E[r]+1-r]$ , that  $|\sigma_t(g)|_{r+t} \geq \frac{1}{\mu_0} |g|_r$ . The conclusion follows.  $\square$

LEMMA 12.6. *With the assumptions and notation of Lemma 12.5, if the map  $\psi$  is a (strongly) hyperbolic forest-map of  $(\Gamma, d_\Gamma)$  then the semi-flow  $(\sigma_t)_{t \in \mathbf{R}^+}$  is (strongly) hyperbolic with respect to any horizontal  $d_\Gamma$ -metric.*

The proof is similar to that of Lemma 12.5.  $\square$

*Proof of Theorem 12.4.* By Lemmas 12.5 and 12.6, a mapping-telescope admits a structure of forest-stack  $(\tilde{X}, f, \sigma_t, \mathcal{H})$  with horizontal metric  $\mathcal{H}$  such that the semi-flow  $(\sigma_t)_{t \in \mathbf{R}^+}$  is a strongly hyperbolic semi-flow with respect to  $\mathcal{H}$ . Hence Theorem 4.4 implies Theorem 12.4.  $\square$

## 13. ABOUT MAPPING-TORUS GROUPS

We first recall the definition of a *hyperbolic endomorphism* of a group introduced by Gromov [19].