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LEMMA 5.4. *Let p be a horizontal geodesic which admits a decomposition in r subpaths p_i such that for some constant $L \geq 0$, for any $i = 1, \ldots, r$, either $|[p_i]_{r+nt_0}|_{r+nt_0} \leq |p_i|_r$ or $L \geq |[p_i]_{r+nt_0}|_{r+nt_0} > |p_i|_r$. Then there exists a constant $C_{5.4}(n, r, L)$, which is increasing in each variable, such that if p is dilated in the future after nt_0, then $|p|_r \leq C_{5.4}(n, r, L)$.*

Proof. We set $n = 1$ in order to simplify the notation; the general case is treated in the same way. Up to permuting the indices, $|[p_i]_{r+t_0}|_{r+t_0} > |p_i|_r$ for $i = 1, \ldots, j$. Since p is dilated in the future after t_0,

$$jL + \sum_{i=j+1}^{r} |p_i|_r \geq \lambda^{t_0} \sum_{i=1}^{r} |p_i|_r .$$

Therefore $|p|_r \leq \frac{jL}{\lambda^{t_0}-1}$. □

5.2 STRAIGHT TELESCOPIC PATHS

DEFINITION 5.5. A *straight* telescopic path is a telescopic path S such that if x, y are any two points in S with $x \in O^+(y) \cup O^-(y)$ then the subpath of S between x and y is equal to the orbit-segment of the semi-flow between x and y.

If S is a path containing a point x, let $S_{x,t} \subset S$ be the maximal subpath of S containing x, whose pulled-tight projection $[S_{x,t}]_{f(x)+t}$ on $f^{-1}(f(x)+t)$ is well defined. The point $\sigma_t(x)$ does not necessarily belong to $[S_{x,t}]_{f(x)+t}$. However there exists a unique point in $[S_{x,t}]_{f(x)+t}$ which minimizes the horizontal distance between $\sigma_t(x)$ and $[S_{x,t}]_{f(x)+t}$. This point is denoted by $\overline{x}_t$. Lemma 5.6 below gives an upper bound, depending on t, for the telescopic distance between x and $\overline{x}_t$.

LEMMA 5.6. *Let S be any straight telescopic path. If t is any non negative real number, there exists a constant $C_{5.6}(t) \geq t$, which increases with t, such that any point $x \in S$ is at telescopic distance smaller than $C_{5.6}(t)$ from the point $\overline{x}_t$ (see above).*

Proof. If $\sigma_t(x) \in [S_{x,t}]_{f(x)+t}$, we set $C_{5.6}(t) = t$. Since S is straight, if $\sigma_t(x) \notin [S_{x,t}]_{f(x)+t}$, x belongs to a cancellation c whose endpoints lie in the past orbits of $\overline{x}_t$. The bounded-cancellation property gives an upper bound on the horizontal length of c. This leads to the conclusion. □