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# COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

# (THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

## *DISCUSSION DOCUMENT FOR THE FOURTEENTH ICMI STUDY*

## APPLICATIONS AND MODELLING IN MATHEMATICS EDUCATION

This *Discussion Document* has been prepared for the forthcoming ICMI Study 14 on *Applications and Modelling in Mathematics Education.* Its purpose is to raise important issues related to the theory and practice of teaching, learning, and assessing mathematical modelling and applications, and to stimulate reactions and contributions to these issues. Based on these reactions and contributions, a limited number of participants (approximately 75) will be invited to a *Study Conference*, which is to take place in February 2004 in Dortmund (Germany). Following the conference, a *Study Volume* will be produced whose content will reflect the state-of-the-art in the topic of applications and modelling in mathematics education, and suggest directions for future developments in research and practice. The authors of this Discussion Document are the members of the *International Programme Committee* for this ICMI Study, who are listed in §4 below.

### 1. Rationale for the Study

In this document we shall use the term '*applications and modelling*' broadly to cover all relations and links between the real world and mathematics.

That applications and modelling has been an important theme in mathematics education can be inferred from the extensive literature on the topic, including material generated from a variety of national and international conferences dedicated to the subject. Two particular series are firstly the successive ICMEs (*International Congresses on Mathematical Education*), within which applications and modelling has been addressed regularly inside working and topic groups; and secondly the series of ICTMAs (*International Conferences on the Teaching of Mathematical Modelling and Applications*), which have been held biennially since 1983. While their Proceedings indicate that applications and modelling may play a more important role in some countries' classrooms than in the past, there still exists a substantial gap between the ideals of educational debate and innovative curricula, on the one hand, and everyday teaching practice on the other. In particular, genuine modelling activities are still rather rare in mathematics lessons.

The focus of work in mathematics education that centres on applications and modelling has exhibited considerable variety. Many activities have had a primary focus on *practice*, e.g. construction and trial of mathematical modelling examples for teaching and examinations, writing of application-oriented textbooks, implementation of applications and modelling in existing curricula or development of innovative, modelling-oriented curricula. Several of these activities contain *research* components as well if, as according to Niss, we consider research as "the posing of genuine, non-rhetorical questions (...) to which no satisfactory answers are known as yet (...) and (...) the undertaking of non-trivial investigations of a systematic, reflective and 'methodologically conscious' nature"[1]) in order to obtain answers to those questions.

So it is not surprising that applications and modelling continues to be a central theme in mathematics education. When dealing with questions of *how* individuals acquire mathematical knowledge, we cannot avoid its relationship to reality, especially the relevance of situated learning (including the problem of dependence on specific contexts). Today mathematical models and modelling have invaded a great variety of disciplines, leaving only a few, if any, fields where mathematical models do not play some role. This impact has been substantially supported and accelerated by the availability of powerful *electronic tools*, such as calculators and computers with their enormous processing and communication capabilities.

In the current OECD PISA Study (*Programme for International Student Assessment*), relations between the real world and mathematics are particularly topical. What is being tested in PISA is '*mathematical literacy*', that is "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to engage in mathematics, in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen."[2])

So in mounting this Study on "Applications and Modelling in Mathematics Education", ICMI takes into account reasons such as the above for addressing the importance of relations between mathematics and the real world, incorporating implications of the contemporary state of the educational debate, and in particular addressing the challenge presented by research and development needs in this field. An important aim of the Study is to identify existing shortcomings, and to stimulate further research and development activities.

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[1]) NISS, M. Issues and problems of research on the teaching and learning of applications and modelling. In: J.F. Matos et al., *Modelling and Mathematics Education*, 72–88. Horwood, Chichester, 2001.

[2]) See the PISA mathematics framework in OECD (ed.) *Measuring Student Knowledge and Skills – A New Framework for Assessment.* OECD, Paris, 1999.

## 2. Framework for the Study

Documenting the state-of-the-art in a field and identifying deficiencies and needed research requires a *structuring framework*. This is particularly important in an area as complex and difficult to survey as the teaching and learning of mathematical modelling and applications, for this topic not only deals with most of the essential aspects of the teaching and learning of mathematics at large, but it also touches upon a wide variety of versions of the real world outside mathematics that one seeks to model.

### 2.1 Concepts and notions

It is recognised that within the field variations exist with respect to the meaning and interpretation of some technical terms. Hence we give some working definitions that will indicate meanings intended within this document.

By *real world* we mean everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific and scholarly disciplines different from mathematics. Starting with a certain problematic *situation* in the real world, simplification and structuring leads to the formulation of a *problem* and thence to a *mathematical model* of the problem. Here we use the term problem to encompass both basic practical problems, and problems of an intellectual nature aimed at describing, explaining, understanding or even designing parts of the world. The situation – still a part of the real world in our sense – is then *mathematised* – that is the relevant objects, data, relations and conditions involved in it are translated into mathematics, resulting in a *mathematical model* of the original situation.

Now mathematical methods are used to derive *mathematical results*, which when re-translated into the real world, can be interpreted in relation to the original situation. At this point the problem solver *evaluates* the model by checking whether the problem solution is appropriate and reasonable for his or her purposes. If need be, as often occurs with 'genuine' real-world problems, the whole process has to be repeated with a modified or a totally different model. Finally the ultimate solution of the original real world problem is stated and communicated. It has become common practice to use the term *mathematical modelling* for the entire process consisting of structuring, mathematising, working mathematically and interpreting, validating, revisiting and reporting the model.

Sometimes the given problem situation is already pre-structured or is essentially a 'dressing up' of a purely mathematical problem in the words of a segment of the real world. This is often the case with classical school *word problems*. In this case mathematising means merely 'undressing' the problem, and the 'modelling process' extends no further than some use of mathematics and a simple interpretation.

An *application* of mathematics is sometimes used to describe any kind of linking of the real world and mathematics, for example beginning with a standard piece of analysis and showing how it can be applied to address a problem in another discipline. This is sometimes described as the *use of standard models*, as distinct from *mathematical modelling* that represents a complete process in the sense described above. During the last decade the term '*applications and modelling*' has been increasingly used to encompass all kinds of relationships between the real world and mathematics.

## 2.2 Structure of the topic Applications and Modelling in Mathematics Education

We can conveniently represent the 'reality' of applications and modelling in mathematics education as being constituted essentially by two dimensions.

The *first* dimension contains three different domains, each forming some sort of a continuum. The *first* domain consists of the essential *notions of applications and modelling*, i.e. what we mean by an application of mathematics, and by mathematical modelling; their most important components in terms of concepts and processes; epistemological characteristics of applications and modelling vis-à-vis mathematics as a discipline and vis-à-vis other disciplines and areas of practice; who uses mathematics, and for what purposes; etc. The *second* domain is that of the *classroom*, used generically to indicate the location of teaching and learning activities pertaining to applications and modelling. This includes the classroom in a literal sense, but also includes the student doing his or her homework, individual or group activity, the teacher's planning of teaching activities or looking at students' products, and so forth. The *third* and final domain is the *system* domain. The word system, here, refers to the whole institutional, political, structural, organisational, administrative, financial, social, and physical environment that exerts an influence on the teaching and learning of applications and modelling.

The *second* dimension is constituted by the different educational *levels* within which the teaching and learning of applications and modelling are addressed. For the sake of generality we have stayed with the widespread designations of *primary*, *secondary*, and *tertiary* levels, together with the level of *teacher education*. If we then think of the dimensions as being depicted by orthogonal axes, this choice of structure enables us to locate issues in terms of the $4 \times 3$ cell matrix defined by the respective domains and levels. To clarify we provide an illustrative example.

Issue 0. An underlying reason for giving prominence to applications and modelling is the goal of students being able to apply mathematical knowledge in situations that are new to them. That is, it is argued that applications and modelling competency if well taught can be transferred to solve unfamiliar problems. However, several research studies suggest that for some students at least this transfer is rather limited in scope and range. A significant question is therefore:

*To what extent is applications and modelling competency transferable across areas and contexts ? What teaching/learning experiences are needed or suitable to foster such transferability ?*

As it stands, this issue concerns the applications and modelling domain, the classroom domain, and at least one of the educational levels. The issue is therefore identified within a rectangle whose base extends across two domains, and whose height is determined by the number of educational levels selected. When each issue is viewed in this way the total distribution of issues can be readily identified in terms of the matrix components, and priorities and gaps identified. In this Discussion Document, a number of issues have been identified as particularly significant to the present Study, and readers are invited to comment on these issues or to suggest further issues to be considered in the Study.

The formulation of the issue given above as an example consists of two parts. Firstly, a background part outlining a challenge, i.e. a dilemma or a problem, which may be of a political, practical, or intellectual nature – we call this part the *challenge* part of the issue. The second part consists of particular question(s) that serve the purpose of pinpointing some crucial aspects of the challenge that deserve to be dealt with in the Study. These are set out in boxes for the respective issues identified in §3.

From the viewpoint of this Study, an issue concerning applications and modelling in mathematics education may be viewed and approached – depending on its nature – from a variety of different *perspectives*, each indicating the category of answers sought.

- *doing:* actual teaching and learning practice (enacted or potential) as carried out in classrooms.
- *development and design:* for example curriculum design, teaching, learning, and assessment materials or activities, and so forth.
- *research:* focus is on the generation of answers to questions as yet unanswered.
- *policy:* focus is on the strategies and policies that exist or ought to be adopted in order to place matters pertaining to applications and modelling on the appropriate agenda.

A given issue may be addressed from any or all of these four perspectives, which are not intended to convey a hierarchy of importance.

## 3. Examples of important issues

In this section a number of selected *issues* – consisting of *challenges* and *questions* – are raised. Although certain inherent features have influenced their grouping, there is overlap between them and different groupings are certainly possible. They are intended as a guide to the kinds of issues that the present Study intends to address, and readers are invited to identify additional relevant issues.

### 3.1 Epistemology

Different characterisations of modelling and applications include: posing and solving open-ended questions, creating, refining and validating models, mathematising situations, designing and conducting simulations, solving word problems and engaging in applied problem solving. These present challenges if individuals are to engage successfully in applications and modelling activities.

Issue 1. *Which aspects of applications and modelling invite further epistemological analysis? How is the relationship between applications and modelling and mathematics best described? What is the relationship between applications and modelling and the world we live in?*

Examples of specific questions that could be addressed here are:

- What are the process components of modelling? What is meant by or involved in each?
- How does our knowledge of applications and modelling accumulate, evolve and change over time?
- What are the various meanings of 'authenticity' in modelling?

### 3.2 Application problems

There exists a plethora of applications and modelling problems and materials for use in mathematics classrooms at various educational levels. These materials range from mere 'dressed up' mathematical problems to authentic problem situations.

Issue 2. *What does research have to tell us about the significance of authenticity to students' acquisition and development of modelling competency ?*

Examples of specific questions:

- What authentic applications and modelling materials are available worldwide ?
- Taking account of teaching objectives and students' personal situations (experience, competence), how can teachers set up authentic applications and modelling tasks ?
- How does the authenticity of problems and materials affect students' ability to transfer acquired knowledge and competencies to other contexts and situations ?

### 3.3 Modelling abilities and competencies

With the teaching and learning of mathematical modelling and applications, many goals and expectations are combined.

Issue 3a. *How can modelling ability and modelling competency be characterised, and how can it be developed over time ?*

Examples of specific questions:

- Can specific subskills and subcompetencies of 'modelling competency' be identified ?
- How can modelling ability be distinguished from general problem solving abilities ?
- Are there identifiable stages in the development of modelling ability ?
- What are characteristic differences between expert modellers and novice modellers ?
- What is the role of pure mathematics in developing modelling ability ?

Issue 3b. *How can modelling in teacher pre-service and in-service education courses be promoted ?*

Examples of specific questions:

- What is essential in a teacher education programme to enable prospective teachers to experience real, non-trivial modelling situations, and hence acquire modelling competencies for purposes of teaching applications and modelling in their professional future ?
- Which training strategies can help teachers develop security in using applications and modelling in their teaching ?

### 3.4 Beliefs, attitudes, and emotions

Beliefs, attitudes and emotions play important roles in the development of critical and creative senses in mathematics.

ISSUE 4. *To what extent does applications and modelling have the potential to provide an environment to support both students and teachers in their development of appropriate beliefs about and attitudes towards mathematics ?*

Examples of specific questions:

- What are the implications of research on the role of beliefs, attitudes and emotions for changing teaching practice and classroom cultures with respect to applications and modelling ?
- What strategies are feasible for in-service teacher education that will address the fear experienced by some teachers when faced with applications and modelling ?

### 3.5 CURRICULUM AND GOALS

It is argued that applications and modelling can make fundamental contributions to the development of students' mathematical competencies.

ISSUE 5a. *What would be an appropriate balance – in terms of attention, time and effort – between applications and modelling activities and other mathematical activities in mathematics classrooms at different educational levels ?*

Examples of specific questions:

- Is it possible – or desirable – to identify a core curriculum in applications and modelling within the general mathematical curriculum ?
- Which applications, models and modelling processes should be included in the curriculum ? Do answers depend on each teacher or should there be some minimal indications in national and state curricula ?
- Is it beneficial to generate specific courses or programs on applications and modelling or is it better to integrate applications and modelling into standard mathematical courses ?

The university level represents a particularly problematic case. Although there are differences between places and countries, university graduates in mathematics embark on a large variety of professional careers, many of which will have links involving applications and modelling – including research mathematicians through their research or teaching responsibilities.

ISSUE 5b. *Should all university graduates in mathematics acquire some applications and modelling experiences as part of their studies ? If so, what kinds of experiences should they be ?*

Concerning general education at the school level, some special questions arise. Mathematics accounts for a large proportion of time in school – this is only justified if mathematics can contribute to general education for life after school.

ISSUE 5c. *How and to what extent can applications and modelling contribute to building up fundamental competencies and to enriching a student's general education ?*

Examples of specific questions:

- What meanings can be given to 'general education', and what is the role of mathematical modelling therein ?

- What is a suitable balance within general education between creating one's own models of real situations and problems, and making judgements about models made by others ?

### 3.6 Modelling pedagogy

The pedagogy of applications and modelling intersects the pedagogy of pure mathematics in a multitude of ways and requires at the same time a variety of practices that are not part of the traditional mathematics classroom. Approaches to teaching applications and modelling vary from the use of traditional methods and course structures, to those that include a variety of innovative teaching practices.

Issue 6. *What are appropriate pedagogical principles and strategies for the development of applications and modelling courses and their teaching ? Are there different principles and strategies for different educational levels ?*

Examples of specific questions:

- What research evidence is available to inform and support the pedagogical design and implementation of teaching strategies for courses with an applications and modelling focus ?
- What criteria are most helpful in selecting methods and approaches suggested by theories of human development and/or learning ?
- What obstacles appear to inhibit changes in classroom culture e.g. the introduction of interactive group work in applications and modelling ?
- What criteria can be used to choose the most desirable option at a particular point within an applications and modelling teaching segment (e.g. whether to use individual and group activity) ?

### 3.7 Sustained implementation

To sustain change in an educational system is a major challenge as it involves and impacts upon many different parties, including politicians, curriculum developers, teachers, teacher educators, and mathematics faculty members at the post secondary level.

Issue 7. *In spite of a variety of existing materials, textbooks, etc., and of many arguments for the inclusion of modelling in mathematics education, why is it that the actual role of applications and mathematical modelling in everyday teaching practice is still rather marginal, for all levels of education ? How can this situation be reversed to ensure that applications and mathematical modelling is integrated and preserved at all levels of mathematics education ?*

Examples of specific questions:

- What are the major impediments and obstacles that have existed to prevent the introduction of applications and mathematical modelling, and how can these be changed ?
- What are the requirements for developing and sustaining a mathematical modelling environment in traditional courses at school or university ?
- How can it be ensured that the mathematical modelling philosophy in curriculum documents is mirrored in classroom practice ?

## 3.8 ASSESSMENT AND EVALUATION

The teaching and learning of mathematics at all levels is closely related to assessment of student achievement. There seem to be many indications that the assessment modes traditionally used in mathematics education are not fully appropriate to assess students' modelling competency.

ISSUE 8a. *What alternative assessment modes are available to teachers, institutions and educational systems that can capture the essential components of modelling competency, and what are the obstacles to their implementation ?*

Examples of specific questions:

- In assessing mathematical modelling as a process (instead of a product) what can be learnt from assessment in the arts, music, etc. ?
- When mathematical modelling is introduced into traditional courses at school or university, how should assessment procedures be adapted ?
- When state or national centralised testing of students is implemented, how do we ensure that mathematical modelling is assessed validly ?
- How does one reliably assess individual contributions and achievement within group activities and projects ?

There is a need to develop specific means of evaluating programmes with an applications and modelling content.

ISSUE 8b. *What evaluation modes are available that can capture the essential features of applications and modelling, especially of integrated courses, programmes and curricula, and what are the obstacles to their implementation ?*

Examples of specific questions:

- In what way do usual evaluation procedures for mathematical programmes carry over to programmes that combine mathematics with applications and modelling ?
- What counts as success when evaluating outcomes from a modelling programme ? For example, what do biologists, economists, industrial and financial planners, medical practitioners, etc., look for in a student's mathematical modelling abilities ? How does one establish whether a student has achieved these capabilities ?

## 3.9 TECHNOLOGICAL IMPACTS

Many technological devices are available today and many of them are highly relevant for applications and modelling. In a broad sense these technologies include calculators, computers, the Internet and computational or graphical software, as well as various kinds of instruments for measuring, experimenting, etc. These devices provide not only increased computational power, but broaden the range of possibilities for approaches to teaching, learning and assessment.

ISSUE 9. *How should technology be used at different educational levels to effectively develop students' modelling abilities and to enrich the students' experience and capability with open-ended mathematical situations in applications and modelling ?*

Examples of specific questions:

- What implications does technology have for the range of applications and modelling problems that can be introduced?
- How is the culture of the classroom influenced by the presence of technological devices? Does technology compromise thinking and reflection or can these be enhanced by technology? In what ways?
- What evidence of successful or failed practice in teaching and learning applications and modelling has been documented as a direct consequence of the introduction of technology?
- With respect to non-affluent countries: can applications and modelling be successfully undertaken without the availability of technology?
- What are the implications of the availability of technology for the design of assessment items and practices?

## 4. Call for contributions to the Study

The Study Conference will be held in Dortmund (Germany) on February 13–17, 2004. Participation in the Study Conference is by invitation only, given on the basis of a submitted contribution, and is limited to approximately 75 people. The Study Volume, to be published in the ICMI Study Series, will contain selected contributions and reports prepared for the conference, as well as on the outcomes of the conference.

The International Programme Committee (IPC) for the Study invites submission of contributions on specific questions, problems or issues related to this Discussion Document. Contributions, in the form of synopses of research papers, discussion papers or shorter responses, may address questions raised above, or questions that arise in response, or further issues relating to the theme of the Study. Submissions should not exceed 6 pages in length and should reach the Programme Chair at the address below (preferably by e-mail) no later than June 15, 2003, but earlier if possible. All submissions must be in English, the language of the conference.

The members of the International Programme Committee for this Study are:

Werner BLUM (University of Kassel, Germany), *Chair of the IPC*, Claudi ALSINA (University of Technology, Barcelona, Spain), Maria Salett BIEMBENGUT (University of Blumenau, Brazil), Nicolas BOULEAU (École Nationale des Ponts et Chaussées, Marne-la-Vallée, France), Jere CONFREY (University of Texas-Austin, USA), Peter GALBRAITH (University of Queensland, Brisbane, Australia), Toshikazu IKEDA (Yokohama National University, Japan), Thomas LINGEFJÄRD (Gothenburg University, Sweden), Eric MULLER (Brock University, St. Catharines, Canada), Mogens NISS (Roskilde University, Denmark), Lieven VERSCHAFFEL (University of Leuven, Belgium), Shangzhi WANG (Capital Normal University, Beijing, China), Bernard R. HODGSON (Université Laval, Québec, Canada), *ex officio, representing the ICMI Executive Committee*, Hans-Wolfgang HENN (University of Dortmund, Germany), *Chair of the Local Organising Committee*.

For further information and submission of contributions, please contact the Chair of the IPC:

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