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Artikel: QUADRICS, ORTHOGONAL ACTIONS AND INVOLUTIONS IN COMPLEX PROJECTIVE SPACES
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THEOREM 4.4. *This* SO(4) *action on* S^7 *satisfies:*

(1) *It is a hyperpolar isometric action of cohomogeneity 1, with space of orbits the interval* $[0, \frac{\pi}{2}]$.

(2) *The two exceptional orbits are both diffeomorphic to* $P^2_{\mathbf{R}} \times S^3$ *and both are minimally embedded in* S^7.

(3) *The principal orbits are diffeomorphic to* $F^3(2,1) \times S^3$.

(4) *The square of the distance functions to the exceptional orbits are both Bott-Morse functions.*

(5) *The union of the two exceptional orbits, both copies of* $P^2_{\mathbf{R}} \times S^3$, *is the Spanier-Whitehead dual of one principal orbit* $F^3(2,1) \times S^3$.

We notice that the action of $\mathrm{SO}(n+1)$ on $\mathbf{C}^{n+1}$ considered in Section 2 also provides, when $n = 3$, an isometric action of cohomogeneity 1 of SO(4) on S^7. However, in this case the two special orbits are the inverse images of the quadric Q and the real projective space $\Pi \cong P^3_{\mathbf{R}}$ under the projection $S^7 \to P^3_{\mathbf{C}}$. So this action is not equivalent to the "twistorial" one given by Theorem 4.4.

REFERENCES

[Ar1] ARNOLD, V.I. On the distribution of ovals of real plane algebraic curves, involutions of four-dimensional manifolds and the arithmetic of integral quadratic forms. *Funct. Anal. Appl. 5* (1971), 169–176.

[Ar2] —— A branched covering of $\mathbf{C}P^2 \to S^4$, hyperbolicity and projective topology. *Siberian Math. J. 29* (1988), 717–726.

[Ar3] —— Topological content of the Maxwell theorem on multipole representation of spherical functions. *Topological Methods in Nonlinear Analysis 7* (1996), 205–217.

[Ar4] —— Relatives of the quotient of the complex projective plane by complex conjugation. *Proc. Steklov Inst. Math. 224* (1999), 46–56.

[AG] ARNOLD, V.I. and A.B. GIVENTAL. Symplectic geometry. In: *Dynamical Systems IV*, Encycl. Math. Sci. *4*, 1–136. Springer, 1990.

[AB] ATIYAH, M.F. and J. BERNDT. Projective planes, Severi varieties and spheres. To appear in *J. Diff. Geom.*

[AW] ATIYAH, M.F. and E. WITTEN. M-Theory dynamics on a manifold of G_2 holonomy. *Adv. Theor. Math. Phys. 6* (2001), 1–106.

[BGS] BALLMANN, W., M. GROMOV and V. SCHROEDER. *Manifolds of Nonpositive Curvature.* Birkhäuser, 1985.

[DR] DUAN, H.B. and E. REES. Functions whose critical set consists of two connected manifolds. In: "Papers in honor of José Adem", *Bol. Soc. Mat. Mexicana (2) 37* (1992), 139–149.

[Eb] EBERLEIN, P. B. *Geometry of Nonpositively Curved Manifolds.* University of Chicago Press, 1966.

[EL] EELLS, J. and L. LEMAIRE. *Two Reports on Harmonic Maps.* World Scientific Publ., River Edge (N. J.), 1995.

[EV] EELLS, J. and A. VERJOVSKY. Harmonic and Riemannian foliations. *Bol. Soc. Mat. Mexicana (3) 4* (1998), 1–12.

[Gh] GHYS, E. Prolongements des difféomorphismes de la sphère. *L'Enseignement Math. (2) 37* (1991), 45–59.

[HL] HSIANG, W. Y. and B. H. LAWSON. Minimal submanifolds of low cohomogeneity. *J. Differential Geom. 5* (1971), 1–38.

[HPTT] HEINTZE, E., R. PALAIS, C.-L. TERNG and G. THORBERGSSON. Hyperpolar actions on symmetric spaces. In: *Geometry, Topology and Physics for Raoul Bott* (S.-T. Yau, ed.), 214–245. International Press, Cambridge, 1995.

[Ko] KOLLROSS, A. A classification of hyperpolar and cohomogeneity one actions. *Trans. Amer. Math. Soc. 354* (2002), 571–612.

[Ku] KUIPER, N. The quotient space of $\mathbf{C}P(2)$ by complex conjugation is the 4-sphere. *Math. Ann. 208* (1974), 175–177.

[LC] LÊ, D. T. and D. CHENIOT. Remarques sur les deux exposés précédents. In: *Singularités à Cargèse*, 253–252. *Astérisque 7–8*, 1973.

[Li] LIBGOBER, A. Homotopy groups of the complements to singular hypersurfaces II. *Ann. of Math. (2) 139* (1994), 117–144.

[Mar] MARIN, A. $\mathbf{C}P^2/\sigma$ ou Kuiper et Massey au pays des coniques. In: *A la recherche de la topologie perdue.* Progress in Mathematics, Vol. *62*, Birkhäuser, 1986.

[Ma1] MASSEY, W. S. The quotient space of the complex projective plane under conjugation is a 4-sphere. *Geometriæ Dedicata 2* (1973), 371–374.

[Ma2] —— Imbeddings of projective planes and related manifolds in spheres. *Indiana Univ. Math. J. 23* (1974), 791–812.

[Mi1] MILNOR, J. *Morse Theory.* Ann. of Math. Study *51*. Princeton Univ. Press, (5^{th} printing) 1973.

[Mi2] —— *Singular Points of Complex Hypersurfaces.* Ann. of Math. Study *61*. Princeton Univ. Press, 1968.

[Mo] MOSTOVOY, J. Algebraic cycles and antiholomorphic involutions on projective spaces. *Bol. Soc. Mat. Mexicana (3) 6* (2000), 151–170.

[Ph] PHAM, F. Formules de Picard-Lefschetz généralisées et ramification des intégrales. *Bull. Soc. Math. France 93* (1965), 333–367.

[Sa] SALAMON, S. Harmonic and holomorphic maps. In: *Lecture Notes in Math. 1164* (ed. by M. Meschiri et al.), 162–224. Springer, 1985.

[SV] SEADE, J. and A. VERJOVSKY. Higher-dimensional complex Kleinian groups. *Math. Ann. 322* (2002), 279–300.

[SW] SPANIER, E. H. and J. H. C. WHITEHEAD. Duality in homotopy theory. *Mathematika 2* (1955), 56–80.

[Va1] VASSILIEV, V. A. A geometric realization of the homologies of classical Lie groups and complexes, S-dual to flag manifolds. *St. Petersburg Math. J.* (formerly *Leningrad Math. J.*) *3* (1992), 809–815.

[Va2] —— Invariants of knots and complements of discriminants. In: *Developments in Mathematics: the Moscow School*, ed. by V.I. Arnold and M. Monastyrsky, 194–250. Chapman and Hall, 1993.

[Wa] WALL, C.T.C. Duality of real projective plane curves: Klein's equation. *Topology 35* (1996), 355–362.

[Ya] YASUKURA, O. A classification of orthogonal transformation groups of low cohomogeneity. *Tsukuba J. Math 10* (1986), 299–326.

[Za] ZARISKI, O. *Algebraic Surfaces.* Springer Verlag, 2nd edition, 1972.

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