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THEOREM 4.4. *This $SO(4)$ action on S^7 satisfies:*

(1) *It is a hyperpolar isometric action of cohomogeneity 1, with space of orbits the interval $[0, \frac{\pi}{2}]$.*

(2) *The two exceptional orbits are both diffeomorphic to $P_{\mathbf{R}}^2 \times S^3$ and both are minimally embedded in S^7 .*

(3) *The principal orbits are diffeomorphic to $F^3(2, 1) \times S^3$.*

(4) *The square of the distance functions to the exceptional orbits are both Bott-Morse functions.*

(5) *The union of the two exceptional orbits, both copies of $P_{\mathbf{R}}^2 \times S^3$, is the Spanier-Whitehead dual of one principal orbit $F^3(2, 1) \times S^3$.*

We notice that the action of $SO(n+1)$ on \mathbf{C}^{n+1} considered in Section 2 also provides, when $n=3$, an isometric action of cohomogeneity 1 of $SO(4)$ on S^7 . However, in this case the two special orbits are the inverse images of the quadric Q and the real projective space $\Pi \cong P_{\mathbf{R}}^3$ under the projection $S^7 \rightarrow P_{\mathbf{C}}^3$. So this action is not equivalent to the “twistorial” one given by Theorem 4.4.

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