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**THEOREM 4.4.** *This  $\mathrm{SO}(4)$  action on  $S^7$  satisfies:*

- (1) *It is a hyperpolar isometric action of cohomogeneity 1, with space of orbits the interval  $[0, \frac{\pi}{2}]$ .*
- (2) *The two exceptional orbits are both diffeomorphic to  $P_{\mathbf{R}}^2 \times S^3$  and both are minimally embedded in  $S^7$ .*
- (3) *The principal orbits are diffeomorphic to  $F^3(2, 1) \times S^3$ .*
- (4) *The square of the distance functions to the exceptional orbits are both Bott-Morse functions.*
- (5) *The union of the two exceptional orbits, both copies of  $P_{\mathbf{R}}^2 \times S^3$ , is the Spanier-Whitehead dual of one principal orbit  $F^3(2, 1) \times S^3$ .*

We notice that the action of  $\mathrm{SO}(n+1)$  on  $\mathbf{C}^{n+1}$  considered in Section 2 also provides, when  $n=3$ , an isometric action of cohomogeneity 1 of  $\mathrm{SO}(4)$  on  $S^7$ . However, in this case the two special orbits are the inverse images of the quadric  $Q$  and the real projective space  $\Pi \cong P_{\mathbf{R}}^3$  under the projection  $S^7 \rightarrow P_{\mathbf{C}}^3$ . So this action is not equivalent to the “twistorial” one given by Theorem 4.4.

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