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shows that $\bar{x}$ commutes with $\bar{x}\bar{y}\bar{x}^{-1}\bar{y}^{-1}$ in the tile path group. Then Theorem 6.1 (a) shows that $\pi(\mathcal{T}) \cong \mathbf{Z}/9\mathbf{Z}$. This means that the tile homotopy group only detects area, modulo 9.

On the other hand, we can easily show that if $\mathcal{T}$ tiles a rectangle, then both sides must be even. Consider the ways that a tile can touch the edge of a rectangle.

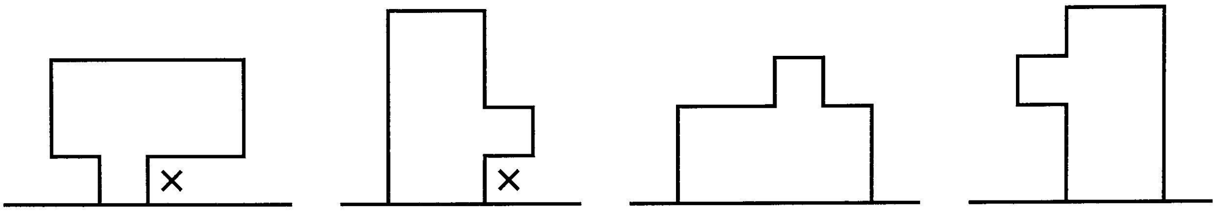
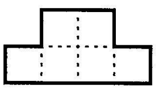


FIGURE 6.9
Tiles along an edge of a rectangle

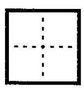
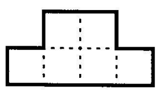
We see that the first two possibilities cannot occur, so each tile that touches the edge does so along an even length. Therefore, each edge of the rectangle has even length. In fact, it is not much harder to show that if $\mathcal{T}$ tiles an $m \times n$ rectangle, then both m and n are multiples of 6. A straightforward argument shows that every tiling of a quadrant by $\mathcal{T}$ is a union of 6×6 squares, which implies the result.

7. APPENDIX: FURTHER EXAMPLES

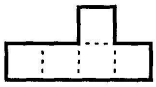
Here we give some more tiling restrictions we have found using the tile homotopy technique. In each case, there are signed tilings that show that the result cannot be obtained by tile homology methods, and there are tilings that show that the result is non-vacuous. Further details will be published elsewhere.

THEOREM 7.1. *Let* $\mathcal{T} = \{$  $\}$, *where all orientations are allowed.*

(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then either* m *or* n *is a multiple of* 4.
(b) *A* 1×6 *rectangle has a signed tiling by* $\mathcal{T}$.

THEOREM 7.2. *Let* $\mathcal{T} = \{$ ,  $\}$, *where all orientations are allowed.*

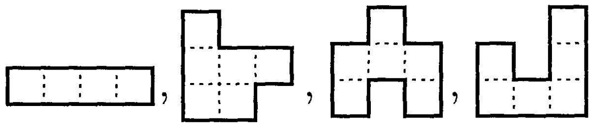
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then* mn *is a multiple of* 4.
(b) *A* 1×6 *rectangle has a signed tiling by* $\mathcal{T}$.

THEOREM 7.3. *Let* $\mathcal{T} = \{$  $\}$, *where rotations are permitted, but reflections are not.*

(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then* mn *is even.*

(b) *A* 1×5 *rectangle has a signed tiling by* $\mathcal{T}$.

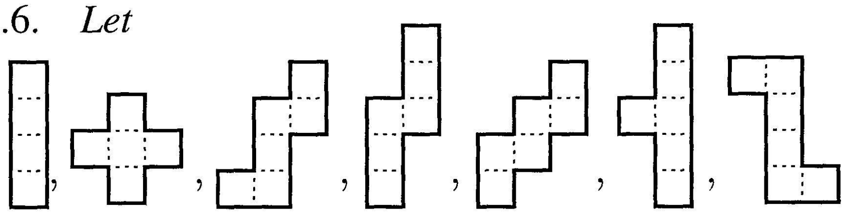
REMARK 7.4. It is easy to show that if $\mathcal{T}$ tiles a rectangle, then both sides are multiples of 5. Also, Yuri Aksyonov [1] has given a clever geometric proof that one side must be a multiple of 10.

THEOREM 7.5. *Let* $\mathcal{T} = \{$  $\}$, *where all orientations are allowed.*

(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 4.

(b) *A* 1×2 *rectangle has a signed tiling by* $\mathcal{T}$.

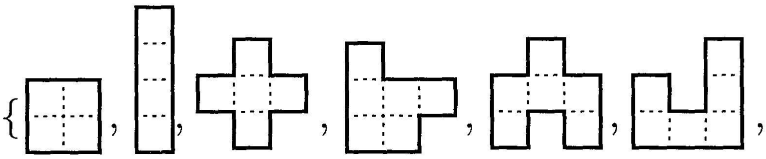
THEOREM 7.6. *Let*

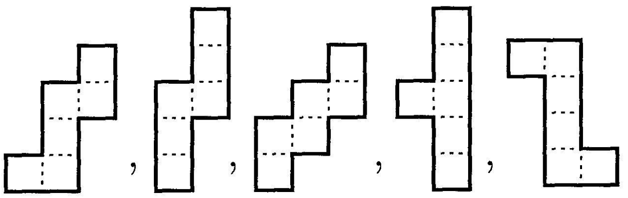
$\mathcal{T} = \{$  $\}$,

where all orientations are allowed.

(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 4.

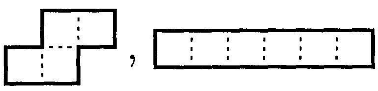
(b) *A* 1×2 *rectangle has a signed tiling by* $\mathcal{T}$.

THEOREM 7.7. *Let* $\mathcal{T} = \{$ 

 $\}$, *where all orientations are allowed.*

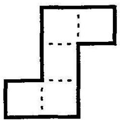
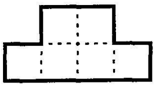
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then* mn *is a multiple of* 4.

(b) *A* 1×2 *rectangle has a signed tiling by* $\mathcal{T}$.

THEOREM 7.8. *Let* $\mathcal{T} = \{$  $\}$, *where all orientations are allowed.*

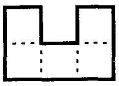
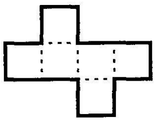
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 6.

(b) *A* 2×2 *square has a signed tiling by* $\mathcal{T}$.

THEOREM 7.9. *Let* $\mathcal{T} = \{$, $\}$, *where all orientations are allowed.*

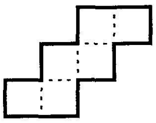
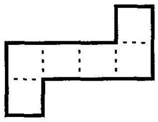
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then either* m *is a multiple of* 3 *or* n *is a multiple of* 6.

(b) *A* 1×1 *square has a signed tiling by* $\mathcal{T}$.

THEOREM 7.10. *Let* $\mathcal{T} = \{$, , $\}$, *where all orientations are allowed.*

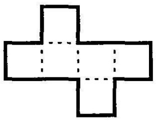
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 8.

(b) *A* 1×1 *square has a signed tiling by* $\mathcal{T}$.

THEOREM 7.11. *Let* $\mathcal{T} = \{$, , $\}$, *where all orientations are allowed.*

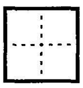
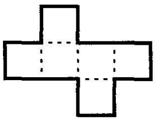
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 5.

(b) *A* 1×1 *square has a signed tiling by* $\mathcal{T}$.

THEOREM 7.12. *Let* $\mathcal{T} = \{$, $\}$, *where all orientations are allowed.*

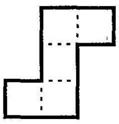
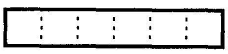
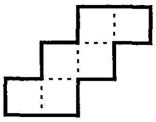
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 4.

(b) *A* 1×2 *rectangle has a signed tiling by* $\mathcal{T}$.

THEOREM 7.13. *Let* $\mathcal{T} = \{$, , $\}$, *where all orientations are allowed.*

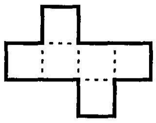
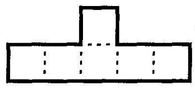
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then* mn *is a multiple of* 4.

(b) *A* 1×2 *rectangle has a signed tiling by* $\mathcal{T}$.

THEOREM 7.14. *Let* $\mathcal{T} = \{$, , $\}$, *where all orientations are allowed.*

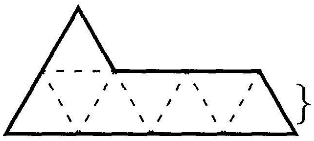
(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 6.

(b) *A* 1×1 *square has a signed tiling by* $\mathcal{T}$.

THEOREM 7.15. *Let* $\mathcal{T} = \{$ , , $\}$, *where all orientations are allowed.*

(a) *If* $\mathcal{T}$ *tiles an* $m \times n$ *rectangle, then one of* m *or* n *is a multiple of* 6.

(b) *A* 2×3 *rectangle has a signed tiling by* $\mathcal{T}$.

THEOREM 7.16. *Let* $\mathcal{T} = \{$ $\}$, *where all orientations are allowed.*

(a) *If* $\mathcal{T}$ *tiles a triangle of side* n, *then* n *is a multiple of* 8.

(b) *A triangle of side* 4 *has a signed tiling by* $\mathcal{T}$.

REMARK 7.17. That $\mathcal{T}$ tiles any triangle is quite interesting. Karl Scherer [15, 2.6 D] has found a tiling of a side 32 triangle by $\mathcal{T}$.

ACKNOWLEDGMENT. I thank Torsten Sillke for some interesting discussions.

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