

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 48 (2002)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE HILBERT METRIC AND GROMOV HYPERBOLICITY

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Bibliographie

DOI: <https://doi.org/10.5169/seals-66068>

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REMARK 5.4. We suggest that a similar statement might hold for the classical Teichmüller spaces and perhaps also for more general Kobayashi hyperbolic complex spaces. Hilbert geodesic rays from a point y that terminate on a line segment contained in the boundary may correspond to the Teichmüller geodesic rays defined by Jenkins-Strebel differentials that H. Masur considered when demonstrating the failure of CAT(0) for the Teichmüller space of Riemann surfaces of genus $g \geq 2$. The complement of the union of all line segments in the boundary ∂D may correspond to the uniquely ergodic foliation points on the Thurston boundary of Teichmüller space.

Using the arguments in [Ka01], see Proposition 5.1 of that paper, we obtain the following result as an application of Theorem 5.2:

THEOREM 5.5. *Let D be a bounded convex domain and $\varphi: D \rightarrow D$ be a map which does not increase Hilbert distances. Then either the orbit $\{\varphi^n(y)\}_{n=1}^\infty$ is bounded or there is a limit point \bar{y} of the orbit such that for any other limit point \bar{x} of the orbit it holds that $[\bar{x}, \bar{y}] \subset \partial D$.*

This theorem, which extends a theorem in [Be97], provides a general geometric explanation for a part of the main theorem in [Me01].

REFERENCES

- [BB00] BALOGH, Z. M. and M. BONK. Gromov hyperbolicity and the Kobayashi metric on strictly pseudoconvex domains. *Comment. Math. Helv.* 75 (2000), 504–533.
- [Be97] BEARDON, A. F. The dynamics of contractions. *Ergodic Theory Dynam. Systems* 17(6) (1997), 1257–1266.
- [Be99] —— The Klein, Hilbert and Poincaré metrics of a domain. *J. Comput. Appl. Math.*, 105(1–2) (1999), 155–162. Continued fractions and geometric function theory (CONFUN), Trondheim, 1997.
- [B00] BENOIST, Y. Convexes divisibles. *C. R. Acad. Sci. Paris Sér. I Math.* 322 (2001), no. 5, 387–390.
- [BG88] BERGER, M. and B. GOSTIAUX. *Differential Geometry: Manifolds, Curves, and Surfaces*. Translated from the French by Silvio Levy. Springer-Verlag, New York, 1988.
- [Bi57] BIRKHOFF, G. Extensions of Jentzsch’s theorem. *Trans. Amer. Math. Soc.* 85 (1957), 219–227.
- [BH99] BRIDSON, M. and A. HAELIGER. *Metric Spaces of Non-Positive Curvature*. Grundlehren der Math. Wissenschaften, 319. Springer-Verlag (Berlin), 1999.

- [Bu55] BUSEMANN, H. *The Geometry of Geodesics*. Academic Press Inc. (New York), 1955.
- [dlH93] DE LA HARPE, P. On Hilbert's metric for simplices. In: *Geometric Group Theory, Vol. 1 (Sussex, 1991)*, 97–119. Cambridge Univ. Press, 1993.
- [Iv97] IVANOV, V.I. A short proof of Gromov non-hyperbolicity of Teichmüller spaces. Preprint, Michigan State University, 1997.
- [Ka01] KARLSSON, A. Nonexpanding maps and Busemann functions. *Ergodic Theory Dynam. Systems* 21 (2001), 1447–1457.
- [KS58] KELLY, P. and E. STRAUS. Curvature in Hilbert geometries. *Pacific J. Math.* 8 (1958), 119–125.
- [Kl78] KLINGENBERG, W. *A Course in Differential Geometry*. Translated from the German by David Hoffman. Graduate Texts in Mathematics, Vol. 51. Springer-Verlag (New York), 1978.
- [Ko84] KOBAYASHI, S. Projectively invariant distances for affine and projective structures. In: *Differential Geometry (Warsaw, 1979)*, 127–152. PWN (Warsaw), 1984.
- [Li95] LIVERANI, C. Decay of correlations. *Ann. of Math.* (2) 142 (1995), 239–301.
- [MW95] MASUR, H. and M. WOLF. Teichmüller space is not Gromov hyperbolic. *Ann. Acad. Sci. Fenn. Ser. A I Math.* 20(2) (1995), 259–267.
- [Me95] METZ, V. Hilbert's projective metric on cones of Dirichlet forms. *J. Funct. Anal.* 127 (1995), 438–455.
- [Me01] —— ‘Laplacians’ on finitely ramified, graph directed fractals. Preprint, 2001.
- [RV73] ROBERTS, A. W. and D. E. VARBERG. *Convex Functions*. Pure and Applied Mathematics, Vol. 57. Academic Press (New York-London), 1973.
- [SM00] SOCIÉ-MÉTHOU, E. Comportements asymptotiques et rigidités en géométries de Hilbert. Thesis, IRMA-Strasbourg, 2000.
- [Sp79] SPIVAK, M. *A Comprehensive Introduction to Differential Geometry*, Vol. 2. Publish or Perish, Inc., 1979.

(Reçu le 30 avril 2001)

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