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is the complex number

$$T(\sigma_1,\sigma_2,\sigma_3) = (1-\sigma_2^*\sigma_1)^{-1}(1-\sigma_2^*\sigma_3)(1-\sigma_1^*\sigma_3)^{-1}\,.$$

The group $\mathrm{GL}(q,\mathbf{C}) \simeq \mathbf{C}^*$ acts on the upper halfplane by $(\lambda, z) \mapsto |\lambda|^2 z$ and so the orbits are described by the argument of the complex number z. So the characteristic invariant in this case is just

$$\arg\left((1-\sigma_2^*\sigma_1)^{-1}(1-\sigma_2^*\sigma_3)(1-\sigma_1^*\sigma_3)^{-1}\right).$$

It is equivalent to the invariant θ considered in [KR]. This invariant, almost in our terms, was known to E. Cartan (see [Ca]).

REMARK 2. Let us consider the case where $p = q$. Then the Stiefel manifold is $\mathrm{U}(q)$, and the content of Proposition 4.2 is that for $(\sigma_1,\sigma_2,\sigma_3) \in S_\top^3$

$$T(\sigma_1,\sigma_2,\sigma_3) = (1-\sigma_2^*\sigma_1)^{-1}(1-\sigma_2^*\sigma_3)(1-\sigma_1^*\sigma_3)^{-1}$$

is an invertible skew-Hermitian matrix. The orbits of $\mathrm{GL}(q,\mathbf{C})$ in its action on nondegenerate Hermitian forms are characterized by the signature. So the characteristic invariant as described in Theorem 4.3 in this case reduces to $\operatorname{sgn} iT(\sigma_1,\sigma_2,\sigma_3)$. As concerns Theorem 4.4, notice that the invariant S is trivial (equal to $-\mathbf{1}_q$), so one is only concerned with the invariant $\arg\det T$. The bounded domain D is of tube type and the description of the invariant through the function $\arg\det$ coincides with the approach of this problem in [CØ], where the invariant was introduced under the name of *generalized Maslov index.*

REFERENCES

[Ca] CARTAN, E. Sur le groupe de la géométrie hypersphérique. *Comment. Math. Helv. 4* (1932), 158–171.

[CØ] CLERC, J.-L. and B. ØRSTED. The Maslov index revisited. *Transform. Groups 6* (2001), 303–320.

[FK] FARAUT, J. and A. KORÁNYI. *Analysis on Symmetric Cones.* Oxford Mathematical Monographs, Clarendon Press, Oxford, 1994.

[Fal] FARAUT, J. et al. *Analysis and Geometry on Complex Homogeneous Domains.* Progress in Mathematics *185*. Birkhäuser Verlag, Boston, 2000.

[H] HUA, L.-K. Geometries of matrices, I. Generalizations of von Staudt's theorem. *Trans. Amer. Math. Soc. 57* (1945), 441–481.

[KR] KORÁNYI, A. and H. M. REIMANN. The complex cross ratio on the Heisenberg group. *L'Enseign. Math. (2) 33* (1987), 291–300.

[P] PIATETSKIĬ-SHAPIRO, L. I. *Geometry of Classical Domains and Theory of Automorphic Forms*. Gordon and Breach, New York, 1969.

[S] SATAKE, I. *Algebraic Structures of Symmetric Domains.* Kanô Memorial Lectures (4). Iwanami Shoten and Princeton University Press, 1980.

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