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one obtains a contradiction because totally geodesic submanifolds are obtained by intersecting the hyperbolic space  $H^n$  with Lorentzian subspaces. Thus,  $G$  must act transitively on each horosphere.

Finally, if  $G$  acts irreducibly then  $G$  must act transitively on the hyperbolic space and must be semisimple of noncompact type by a previous observation. Then, showing that the isotropy group at some point agrees with a maximal compact subgroup, the second part of the theorem follows from the theory of Riemannian symmetric spaces of noncompact type [He].

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