

Zeitschrift:	L'Enseignement Mathématique
Herausgeber:	Commission Internationale de l'Enseignement Mathématique
Band:	48 (2002)
Heft:	3-4: L'ENSEIGNEMENT MATHÉMATIQUE
 Artikel:	THE NONAMENABILITY OF SCHREIER GRAPHS FOR INFINITE INDEX QUASICONVEX SUBGROUPS OF HYPERBOLIC GROUPS
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Kapitel:	3. Hyperbolic metric spaces
DOI:	https://doi.org/10.5169/seals-66081

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3. HYPERBOLIC METRIC SPACES

We refer the reader to [1, 4, 14, 20, 25, 32, 40] for the basic information about Gromov-hyperbolic metric spaces. We briefly recall the main definitions.

If (X, d) is a geodesic metric space and $x, y \in X$, we shall denote by $[x, y]$ a geodesic segment from x to y in X .

DEFINITION 3.1 (Gromov product). Let (X, d) be a metric space and suppose $x, y, z \in X$. We set

$$(x, y)_z := \frac{1}{2}[d(z, x) + d(z, y) - d(x, y)].$$

Note that $(x, y)_z = (y, x)_z$.

DEFINITION 3.2 (Hyperbolic metric space [1]). Let (X, d) be a geodesic metric space. We say that (X, d) is δ -hyperbolic (where $\delta \geq 0$) if for any $p, x, y, z \in X$ we have:

$$(x, y)_p \geq \min\{(x, z)_p, (y, z)_p\} - \delta.$$

The space X is said to be *hyperbolic* if it is δ -hyperbolic for some $\delta \geq 0$.

There are many equivalent definitions of hyperbolicity, for example:

PROPOSITION 3.3 ([1, 20, 32]). *Let (X, d) be a geodesic metric space. Then the following conditions are equivalent.*

1. *The space X is hyperbolic.*
2. *There exists a constant $\delta' \geq 0$ such that if $x, y, z \in X$ and $y' \in [x, y]$, $z' \in [x, z]$ are such that $d(x, y') = d(x, z') \leq (y, z)_x$ then $d(y', z') \leq \delta'$.*
3. *(Thin Triangles Condition) There exists $\delta'' \geq 0$ such that for any $x, y, z \in X$, for any geodesic segments $[x, y]$, $[x, z]$ and $[y, z]$ and for any point $p \in [x, y]$ there is a point $q \in [x, z] \cup [y, z]$ such that $d(p, q) \leq \delta''$.*

DEFINITION 3.4 (Word-hyperbolic group). A finitely generated group G is said to be *word-hyperbolic* if for some (and hence for any) finite generating set A of G the Cayley graph $\Gamma(G, A)$ is hyperbolic.

DEFINITION 3.5 (Gromov product for sets). Let (X, d) be a metric space. Let $x \in X$ and $Q, Q' \subseteq X$. Define $(Q, Q')_x := \sup\{(q, q')_x \mid q \in Q, q' \in Q'\}$.