Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 48 (2002)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: PROJECTIVE MODULES OVER SOME PRÜFER RINGS
Autor: Feng, L. G. / Lam, T. Y.
Bibliographie
DOI: https://doi.org/10.5169/seals-66080

where, for each $i' \in \Lambda'$, $P(i') := \bigoplus_{n \in \mathbf{N}} P_{(n,i')}$. By Lemma 11, each $P(i')$ is free, so by (13), P is also free. This proves the first part of the theorem.

For the second part, let P be any *non countably generated* projective R-module. By Kaplansky's theorem in [Ka$_3$], we can express P in the form $\bigoplus_{i \in \Lambda} P_i$ (for some indexing set Λ), where the P_i's are nonzero countably generated projective R-modules. Since P itself is not countably generated, Λ must be an infinite set. Thus, the first part of the theorem applies, showing that P is free. □

It seems plausible that, under the assumptions on R in Theorem 12, any countably but not finitely generated projective R-module P is also free. This would follow from Lemma 11 if we can decompose P as in that Lemma. However, we are not able to prove the existence of such a decomposition.

We close by recalling that most results in this note required the small 0-divisor assumption on R. The study of projective modules over general Prüfer rings (without the small 0-divisor assumption) awaits further effort.

REFERENCES

[AP] ANDERSON, D. D. and J. PASCUAL. Regular ideals in commutative rings, sublattices of regular ideals, and Prüfer rings. *J. Algebra 111* (1987), 404–426.

[Ba] BASS, H. Projective modules over free groups are free. *J. Algebra 1* (1964), 367–373.

[BK] BREWER, J. and L. KLINGLER. Pole assignability and the invariant factor theorem in Prüfer domains and Dedekind domains. *J. Algebra 111* (1987), 536–545.

[CE] CARTAN, H. and S. EILENBERG. *Homological Algebra.* Princeton Univ. Press, 1956.

[FHP] FONTANA, M., J. A. HUCKABA and I. J. PAPICK. *Prüfer Domains.* Monographs in Pure and Applied Math., Vol. 203. Marcel Dekker, New York–Basel–Hong Kong, 1997.

[FS] FUCHS, L. and L. SALCE. *Modules over Non-Noetherian Domains.* Math. Surveys and Monographs, Vol. 84. Amer. Math. Soc., Providence (R.I.), 2001.

[GH] GILMER, R. and W. HEINZER. On the number of generators of an invertible ideal. *J. Algebra 14* (1970), 139–151.

[Gr] GRIFFIN, M. Prüfer rings with zero divisors. *J. reine angew. Math. 239–240* (1970), 55–67.

[HL] HEITMANN, R. C. and L. S. LEVY. $1\frac{1}{2}$ and 2 generator ideals in Prüfer domains. *Rocky Mountain J. Math. 5* (1975), 361–373.

[Ka1] KAPLANSKY, I. Elementary divisors and modules. *Trans. Amer. Math. Soc. 66* (1949), 464–491.

[Ka2] —— Modules over Dedekind rings and valuation rings. *Trans. Amer. Math. Soc. 72* (1952), 327–340.

[Ka3] —— Projective modules. *Ann. of Math. (2) 68* (1958), 372–377.

[Ka4] —— *Commutative Rings.* Revised edition. Univ. of Chicago Press, Chicago and London, 1974.

[La1] LAM, T. Y. *A First Course in Noncommutative Rings.* Second edition, Graduate Texts in Math., Vol. 131. Springer-Verlag, Berlin–Heidelberg–New York, 2001.

[La2] —— *Lectures on Modules and Rings.* Graduate Texts in Math., Vol. 189. Springer-Verlag, Berlin–Heidelberg–New York, 1998.

[LM] LARSEN, M. D. and P. J. MCCARTHY. *Multiplicative Theory of Ideals.* Monographs in Pure and Applied Math., Vol. 43. Academic Press, New York, 1971.

[Sch] SCHÜLTING, H.-W. Über die Erzeugendenanzahl invertierbarer Ideale in Prüferringen. *Comm. Algebra 7* (1979), 1331–1349.

(Reçu le 18 avril 2002)

L. G. Feng

National University of Defense Technology
Changsha 410073
P. R. China
e-mail: flg@math.berkeley.edu

T. Y. Lam

University of California
Berkeley, CA 94720
U. S. A.
e-mail: lam@math.berkeley.edu

vide-leer-empty