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TORSION NUMBERS OF AUGMENTED GROUPS WITH APPLICATIONS TO KNOTS AND LINKS

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Dedicated to the memory of Arnold E. Ross

ABSTRACT. Torsion and Betti numbers for knots are special cases of more general invariants b_r and β_r, respectively, associated to a finitely generated group G and epimorphism $\chi\colon G \to \mathbf{Z}$. The sequence of Betti numbers is always periodic; under mild hypotheses about (G, χ), the sequence b_r satisfies a linear homogeneous recurrence relation with constant coefficients. Generally, b_r exhibits exponential growth rate. However, again under mild hypotheses, the p-part of b_r has trivial growth for any prime p. Applications to branched cover homology for knots and links are presented.

1. INTRODUCTION

A *knot* is a simple closed curve in the 3-sphere S^3. Knots are *equivalent* if there is an orientation-preserving homeomorphism of S^3 that carries one into the other. Equivalent knots are regarded as the same. An *invariant* is a well-defined quantity that depends only on a knot equivalence class. Two knots for which some invariant differs are necessarily distinct.

Associated to any knot k and natural number r there is a compact, oriented 3-manifold M_r, the r-fold cyclic cover of S^3 branched over k. A precise definition can be found in [Li97] or [Ro76], for example. Topological invariants of M_r are invariants of k. Two such invariants, the first Betti number β_r and the order b_r of the torsion subgroup of $H_1(M_r; \mathbf{Z})$, were first considered by J. Alexander and G. Briggs [Al28], [AB27] and by O. Zariski [Za32]. The continuing interest in these invariants is witnessed by numerous papers (e.g., [Go72], [Me80], [We80], [Ri90] and [GS91]). We call b_r the r^{th} *torsion*

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