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iii) If X comes with an orientation, we may assume that D^m is orientation preservingly embedded and that all attaching maps are orientation reversing.

3.2 CONSEQUENCES FOR E-MANIFOLDS OF DIMENSION EIGHT WITH $w_2 = 0$

Let X be a piecewise linear (smooth) E-manifold of dimension eight with $w_2(X) = 0$. The first observation concerns the structure of W_2 .

LEMMA 3.5. *One has $W_2 \cong \#_{i=1}^b(S^2 \times D^6)$.*

Proof. The manifold W_2 is an $(8, 1)$ -handle body and as such homeomorphic to the boundary connected sum of b D^6 -bundles over S^2 ([17], §11, p.115). As $\pi_1(\text{SO}(4)) \cong \mathbf{Z}_2$ and we have requested $w_2(X) = 0$, the claim follows. \square

The next consequence is

The manifold W_4 is determined by a framed link of $b_4(X)$ three-dimensional spheres in $\#_{i=1}^b(S^2 \times S^5)$.

We shall look into the classification of such links below. The third consequence is

LEMMA 3.6. i) $\partial W_4 \cong \#_{i=1}^b(S^2 \times S^5)$.

ii) *The manifold X is of the form $W_4 \cup_f \#_{i=1}^b(S^2 \times D^6)$ where*

$$f: \#_{i=1}^b(S^2 \times S^5) \longrightarrow \partial W_4$$

is a piecewise linear (smooth) isomorphism, such that f_ maps the canonical basis of $H_2(\#_{i=1}^b(S^2 \times S^5), \mathbf{Z})$ to the preferred basis of $H_2(\partial W_4, \mathbf{Z})$.*

Proof. i) This follows because $\partial W_4 \cong \partial \bar{W}_2$. ii) follows because $X = W_4 \cup \bar{W}_2$, and $\bar{W}_2 \cong \#_{i=1}^b(S^2 \times D^6)$, by Lemma 3.5. \square

3.3 HOMOTOPY VS. ISOTOPY

By Theorem 3.3, the manifold is determined by the ambient isotopy classes of the attaching maps. However, the topological invariants of the manifold give us at best their homotopy classes. It is, therefore, important to have theorems granting that this is enough. In the setting of differentiable manifolds, we have