Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 48 (2002)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE CLASSIFICATION OF CERTAIN PIECEWISE LINEAR AND DIFFERENTIABLE MANIFOLDS IN DIMENSION EIGHT AND AUTOMORPHISMS OF $\sharp_{i=1}^b (S^2 \times S^5)$
Autor: Schmitt, Alexander
Kapitel: 1. Introduction
DOI: https://doi.org/10.5169/seals-66077

Nutzungsbedingungen
Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation
L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use
The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 10.12.2025

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

L'Enseignement Mathématique, t. 48 (2002), p. 263–289

ON THE CLASSIFICATION OF CERTAIN PIECEWISE LINEAR AND DIFFERENTIABLE MANIFOLDS IN DIMENSION EIGHT AND AUTOMORPHISMS OF $\#_{i=1}^{b}(S^2 \times S^5)$

by Alexander SCHMITT

ABSTRACT. In this paper, we will be concerned with the explicit classification of closed, oriented, simply-connected spin manifolds in dimension eight with vanishing cohomology in the odd dimensions. The study of such manifolds was begun by Stefan Müller. In order to understand the structure of these manifolds, we will analyze their minimal handle presentations and describe explicitly to what extent these handle presentations are determined by the cohomology ring and the characteristic classes. It turns out that the cohomology ring and the characteristic classes do not suffice to reconstruct a manifold of the above type completely. In fact, the group $\mathrm{Aut}_0\left(\#_{i=1}^{b}(S^2 \times S^5)\right) / \mathrm{Aut}_0\left(\#_{i=1}^{b}(S^2 \times D^6)\right)$ of automorphisms of $\#_{i=1}^{b}(S^2 \times S^5)$ which induce the identity on cohomology modulo those which extend to $\#_{i=1}^{b}(S^2 \times D^6)$ acts on the set of oriented homeomorphy classes of manifolds with fixed cohomology ring and characteristic classes, and we will be also concerned with describing this group and some facts about the above action.

1. INTRODUCTION

The classification of topological manifolds up to homeomorphy is an extremely interesting and important problem. Let us restrict our attention to the case of closed (i.e., compact without boundary) and oriented simply connected manifolds. As a general classification scheme, surgery theory [1] solves this problem for manifolds within a given homotopy type, e.g., that of a sphere. Another approach to the classification "up to finite indeterminacy", using rational homotopy theory, is due to Sullivan [34]. Nevertheless, there are only a few explicit results which characterize the oriented homeomorphy type of a manifold in terms of easily computable invariants. They usually require many simplifying assumptions, such as high connectedness [36]. In this paper, we will

consider *even cohomology manifolds* (or *E-manifolds*, for short) in dimension eight, by which we mean closed, oriented, simply connected, piecewise linear or smooth manifolds all of whose odd-dimensional homology groups with integer coefficients vanish. The universal coefficient theorem implies that all homology groups of an E-manifold are without torsion. Moreover, since $H^3(X, \mathbf{Z}_2) = 0$ for an E-manifold, two E-manifolds of dimension at least 6 are homeomorphic (as topological manifolds) if and only if they are piecewise linearly homeomorphic [16].

Though the class of E-manifolds is fairly restricted, it still contains many interesting examples from various areas of mathematics, such as the piecewise linear manifolds underlying the toric manifolds from Algebraic Geometry [4] or the polygon spaces [12], to mention a few. So far, E-manifolds have been classified up to dimension 6. Of course, in dimension 2 there is only S^2, in dimension 4, there is the famous classification result of Freedman various interesting aspects of which are discussed in [15], and finally in dimension 6, the classification was achieved by Wall [37] and Jupp [14]. Various applications of the latter result to Algebraic Geometry are surveyed in [26]. Finally, we refer to [2], [3], and [29] for the determination of projective algebraic structures on certain 6- and 8-dimensional E-manifolds.

ACKNOWLEDGEMENTS. My thanks go to J.-C. Hausmann for his interest in this work and for pointing out to me that an E-manifold of dimension eight is actually not determined by its classical invariants.

2. STATEMENT OF THE RESULT

We now discuss the main result of this note, namely the classification of E-manifolds of dimension 8 with vanishing second Stiefel-Whitney class in the form of an exact sequence of pointed sets. This result was motivated by the work [24]. In order to state it in a more elegant form, we will work with based manifolds. By a *based piecewise linear (smooth) E-manifold*, we mean a triple $(X, \underline{x}, \underline{y})$, consisting of a piecewise linear (smooth) E-manifold X and bases $\underline{x} = (x_1, \ldots, x_{b_2(M)})$ for $H^2(X, \mathbf{Z})$ and $\underline{y} = (y_1, \ldots, y_{b_4(M)})$ for $H^4(X, \mathbf{Z})$. Recall that by definition E-manifolds are oriented, so that the above data specify a basis for $H^*(X, \mathbf{Z})$, such that the bases for $H^i(X, \mathbf{Z})$ and $H^{8-i}(X, \mathbf{Z})$ are dual to each other with respect to the cup product. An *isomorphism between piecewise linear (smooth) based E-manifolds* $(X, \underline{x}, \underline{y})$ and $(X', \underline{x}', \underline{y}')$ is an orientation preserving piecewise linear (smooth) isomorphism $f\colon X \longrightarrow X'$ with $f^*(\underline{x}') = \underline{x}$