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ON THE CLASSIFICATION OF CERTAIN PIECEWISE LINEAR AND DIFFERENTIABLE MANIFOLDS IN DIMENSION EIGHT AND AUTOMORPHISMS OF $\#_{i=1}^{b}(S^2 \times S^5)$

by Alexander SCHMITT

ABSTRACT. In this paper, we will be concerned with the explicit classification of closed, oriented, simply-connected spin manifolds in dimension eight with vanishing cohomology in the odd dimensions. The study of such manifolds was begun by Stefan Müller. In order to understand the structure of these manifolds, we will analyze their minimal handle presentations and describe explicitly to what extent these handle presentations are determined by the cohomology ring and the characteristic classes. It turns out that the cohomology ring and the characteristic classes do not suffice to reconstruct a manifold of the above type completely. In fact, the group $\mathrm{Aut}_0\big(\#_{i=1}^{b}(S^2 \times S^5)\big) / \mathrm{Aut}_0\big(\#_{i=1}^{b}(S^2 \times D^6)\big)$ of automorphisms of $\#_{i=1}^{b}(S^2 \times S^5)$ which induce the identity on cohomology modulo those which extend to $\#_{i=1}^{b}(S^2 \times D^6)$ acts on the set of oriented homeomorphy classes of manifolds with fixed cohomology ring and characteristic classes, and we will be also concerned with describing this group and some facts about the above action.

1. INTRODUCTION

The classification of topological manifolds up to homeomorphy is an extremely interesting and important problem. Let us restrict our attention to the case of closed (i.e., compact without boundary) and oriented simply connected manifolds. As a general classification scheme, surgery theory [1] solves this problem for manifolds within a given homotopy type, e.g., that of a sphere. Another approach to the classification "up to finite indeterminacy", using rational homotopy theory, is due to Sullivan [34]. Nevertheless, there are only a few explicit results which characterize the oriented homeomorphy type of a manifold in terms of easily computable invariants. They usually require many simplifying assumptions, such as high connectedness [36]. In this paper, we will