

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 48 (2002)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: MM-SPACES AND GROUP ACTIONS
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Kapitel: 6.2 Dvoretzky's theorem
DOI: <https://doi.org/10.5169/seals-66074>

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DEFINITION 11 [Gr1]. Say that a G -space X (in our agreed sense) has the *Ramsey–Dvoretzky–Milman property* if for every bounded uniformly continuous function f from X to a finite-dimensional Euclidean space, every $\varepsilon > 0$, and every finite $F \subseteq X$, there is a $g \in G$ with the property

$$\text{Osc}(f|_{gF}) < \varepsilon.$$

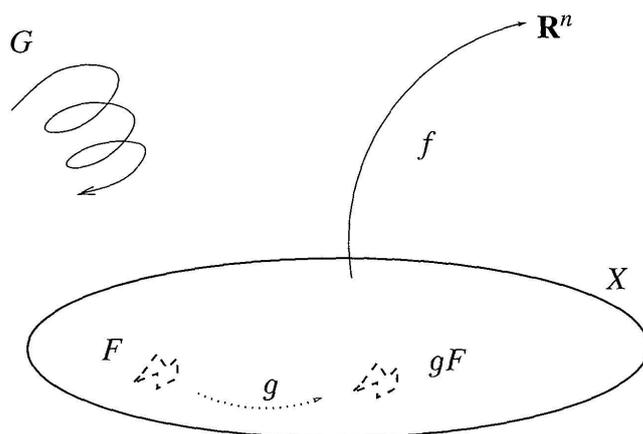


FIGURE 3

The Ramsey–Dvoretzky–Milman property

REMARK 9. Equivalently, F can be assumed compact.

COROLLARY 7. For a topological group G the following are equivalent:

- (i) G is extremely amenable;
- (ii) every metric space X upon which G acts continuously and transitively by isometries has the R–D–M property;
- (iii) every homogeneous factor-space G/H , equipped with a left-invariant metric (or the left uniform structure), has the R–D–M property.

Next, we will discover two very important situations where the R–D–M property appears naturally.

6.2 DVORETZKY'S THEOREM

Here is the famous result.

THEOREM (Arieh Dvoretzky). For all $\varepsilon > 0$ there is a constant $c = c(\varepsilon) > 0$ such that for any n -dimensional normed space $(X, \|\cdot\|_E)$ there is a subspace V of $\dim V \geq c \log n$ and a Euclidean norm $\|\cdot\|_2$ with $\|x\|_2 \leq \|x\|_E \leq (1 + \varepsilon)\|x\|_2$ for all $x \in V$.

The studies of the phenomenon of concentration of measure were given a boost by Vitali Milman's new proof of the Dvoretzky theorem [M1], based on a suitable finite-dimensional approximation to the lemma which follows directly from results that we have previously stated:

LEMMA (Milman). *The pair $(U(\mathcal{H}), \mathbf{S}^\infty)$ has the R-D-M property, where \mathbf{S}^∞ is the unit sphere of an infinite-dimensional Hilbert space \mathcal{H} .*

6.3 RAMSEY'S THEOREM

Let r be a positive natural number. By $[r]$ one denotes the set $\{1, 2, \dots, r\}$. A *colouring* of a set X with r colours, or simply *r-colouring*, is any map $\chi: X \rightarrow [r]$. A subset $A \subseteq X$ is *monochromatic* if for every $a, b \in A$ one has $\chi(a) = \chi(b)$.

Put otherwise, a finite colouring of a set X is nothing but a partition of X into finitely many (disjoint) subsets.

Let X be a set, and let k be a natural number. Denote by $[X]^k$ the set of all k -subsets of X , that is, all (unordered!) subsets containing exactly k elements.

INFINITE RAMSEY THEOREM. *Let X be an infinite set, and let k be a natural number. For every finite colouring of $[X]^k$ there exists an infinite subset $A \subseteq X$ such that the set $[A]^k$ is monochromatic.*

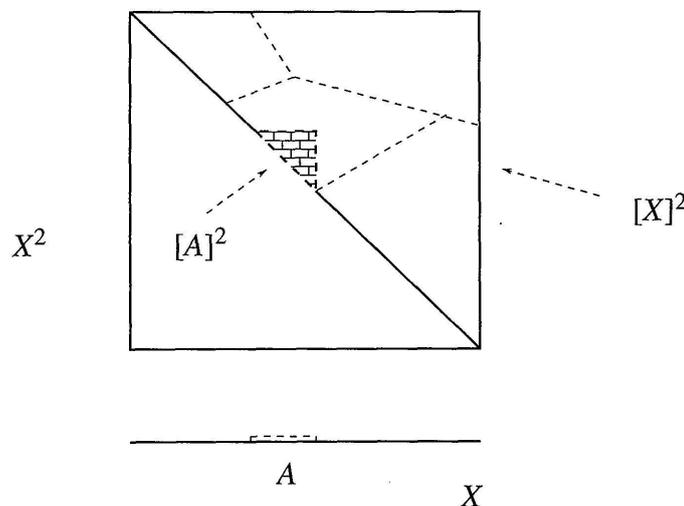


FIGURE 4

Ramsey theorem for $k = 2$