

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 48 (2002)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE RATIONAL FORMS OF NILPOTENT LIE ALGEBRAS AND LATTICES IN NILPOTENT LIE GROUPS
Autor: Semenov, Yu. S.
Kurzfassung
DOI: <https://doi.org/10.5169/seals-66073>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 17.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

ON THE RATIONAL FORMS OF NILPOTENT LIE ALGEBRAS AND LATTICES IN NILPOTENT LIE GROUPS

by Yu. S. SEMENOV*)

ABSTRACT. We study the rational forms of real finite-dimensional nilpotent Lie algebras and the corresponding lattices in nilpotent Lie groups. We show that for some Lie algebras there are infinitely many such rational forms up to isomorphism and give a description of isomorphism classes in several 6-dimensional cases. Nilpotent Lie algebras with a unique rational form are also considered.

1. INTRODUCTION

Let \mathfrak{g} be a finite-dimensional Lie algebra over \mathbf{R} and \mathfrak{h} be a \mathbf{Q} -subalgebra of \mathfrak{g} . We say that \mathfrak{h} is a rational form (or \mathbf{Q} -form) of \mathfrak{g} if there exists a basis X of \mathfrak{h} over \mathbf{Q} such that X is a basis of \mathfrak{g} over \mathbf{R} . In other words, the inclusion $\mathfrak{h} \hookrightarrow \mathfrak{g}$ gives rise to an isomorphism $\mathfrak{h} \otimes_{\mathbf{Q}} \mathbf{R} \cong \mathfrak{g}$.

In the sequel all Lie algebras are assumed to be nilpotent and finite-dimensional unless otherwise specified. The main purpose of the present work is to describe rational forms for some real nilpotent Lie algebras. The rational forms (or their isomorphism classes) in such algebras are closely related to lattices, i.e., discrete cocompact subgroups in nilpotent Lie groups.

Let G be a nilpotent connected 1-connected Lie group and \mathfrak{g} be the Lie algebra of G . It is well known that $\exp: \mathfrak{g} \rightarrow G$ and $\log: G \rightarrow \mathfrak{g}$ are two reciprocal diffeomorphisms. Let \mathfrak{h} be a rational form of \mathfrak{g} and $X = \{x_1, \dots, x_d\}$ be a basis of \mathfrak{h} . Malcev showed in [5] that the subgroup Γ of G generated by $\exp(rx_1), \dots, \exp(rx_d)$ (where r is an appropriate integer) is a lattice of G .

*) This work has been supported by the Swiss National Science Foundation.