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PROJECTIVE GEOMETRY OF POLYGONS  
AND DISCRETE 4-VERTEX AND 6-VERTEX THEOREMS

by V. OVSIENKO and S. TABACHNIKOV

ABSTRACT. This paper is concerned with discrete versions of three well-known results from projective differential geometry: the four-vertex theorem, the theorem on six affine vertices, and Ghys' theorem on four zeroes of the Schwarzian derivative. We study the geometry of closed polygonal lines in  $\mathbf{RP}^d$  and prove that polygons satisfying a certain convexity condition have at least  $d + 1$  flattenings. This result provides a new approach to the classical theorems mentioned.

1. INTRODUCTION

A *vertex* of a smooth plane curve is a point where the curve has 4<sup>th</sup>-order contact with a circle (at a generic point the osculating circle has 3<sup>rd</sup>-order contact with the curve). An *affine vertex* (or *sextactic point*) of a smooth plane curve is a point of 6<sup>th</sup>-order contact with a conic. In 1909 S. Mukhopadhyaya [10] published two celebrated theorems, which are joined in the following statement:

*Every closed smooth convex plane curve has at least 4 distinct vertices and at least 6 distinct affine vertices.*

These results generated an extensive literature. From a modern point of view they are related, among other subjects, to the global singularity theory of wave fronts and to Sturm theory – see e.g. [1, 2, 4, 8, 17, 18] and references therein.