

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 47 (2001)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE POSITIVE CONE OF SPHERES AND SOME PRODUCTS OF SPHERES
Autor: MATTHEY, Michel / SUTER, Ulrich
Kapitel: 10. "Gaps in cohomology" and the -cone
DOI: <https://doi.org/10.5169/seals-65432>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 17.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

REMARK 9.6.

i) This shows that $[x_1, x_2]$ generates $\pi_{11}(BU(3)) \cong \mathbf{Z}/30$ and that the map $j_*: \pi_{11}(BU(3)) \rightarrow \pi_{11}(BU(5))$ is injective.

ii) We were also able to prove this theorem without appealing to results on homotopy groups of $BU(n)$. Using spectral sequence arguments, we have computed the first few stages of the Moore-Postnikov tower of the map $BSU(3) \rightarrow BSU(5)$. This computation, being extremely lengthy, is not given here (see [Matt]).

Now we move on to the product $S^6 \times S^8$.

THEOREM 9.7. *For the product $S^6 \times S^8$, one has*

$$K_+(S^6 \times S^8) = K_c(S^6 \times S^8) = K_\gamma(S^6 \times S^8).$$

The latter is described in Theorem 7.1.

Proof. By Lundell's tables [Lun] (see also [Mim]), one has

$$\pi_{13}(BU(4)) \cong \mathbf{Z}/60 \quad \text{and} \quad \pi_{13}(BU(6)) \cong \mathbf{Z}/720.$$

Corollary 8.3 shows that $[x_1, x_2]$ is of order 60 in $\pi_{13}(BU(6))$. By naturality, the map $j_* = \pi_{13}(j)$, induced by $j: BU(4) \rightarrow BU(6)$, takes the Whitehead product $[x_1, x_2] \in \pi_{13}(BU(4))$ to $[x_1, x_2] \in \pi_{13}(BU(6))$. This implies that $[x_1, x_2]$ is of order 60 in $\pi_{13}(BU(4))$ too, and that $[ax_1, bx_2]$ vanishes in $\pi_{13}(BU(4))$ precisely when it is zero in $\pi_{13}(BU(6))$. Together with Theorem 8.2, this completes the proof. \square

REMARK 9.8. The proof shows that $[x_1, x_2]$ is a generator of the group $\pi_{13}(BU(4)) \cong \mathbf{Z}/60$ and that the map $j_*: \pi_{13}(BU(4)) \rightarrow \pi_{13}(BU(6))$ is injective.

10. "GAPS IN COHOMOLOGY" AND THE γ -CONE

In the present section, we are interested in spaces having a "gap in cohomology", more precisely we look at spaces obtained by attaching a single large-dimensional cell to a finite CW-complex Y . For such spaces, the integral cohomology is zero between the dimension of Y and the top-dimensional class. The products $S^n \times S^m$ are typical examples (see Section 8). For this kind of spaces, the c -cone obviously cannot give information in the

dimensions corresponding to the gap. At first sight, one could think that the γ -cone is more powerful in this range. Unfortunately, this is not the case: we show that the γ -cone (or equivalently the γ -dimension function) is also “blind” in some sense. Here is the precise statement.

PROPOSITION 10.1. *Let Y be a connected finite CW-complex of dimension $\leq 2n$, and let $X = C_f = Y \cup_f e^{2n+2m}$ be the mapping cone of a map $f: S^{2n+2m-1} \rightarrow Y$, with $m \geq 1$. Then, for $x \in \tilde{K}(X)$, one has*

$$\gamma^{n+m}(x) = 0 \implies \gamma^{n+l}(x) = 0 \text{ for all } l = 1, \dots, m.$$

In other words, if $\gamma\text{-dim}(x) < n + m$, then $\gamma\text{-dim}(x) \leq n$.

Proof. By assumption, one has $H^k(X; \mathbf{Z}) = 0$ for $2n < k < 2n + 2m$ and $H^{2n+2m}(X; \mathbf{Z}) \cong \mathbf{Z}$. Let $x \in \tilde{K}(X)$ such that $\gamma^{n+m}(x) = 0$. By Proposition 2.2, keeping the same notation, we have

$$ch(\gamma^k(x)) = \bar{c}_k(x) + P_{k+1}(\bar{c}_1(x), \dots, \bar{c}_{n+m}(x)),$$

and $0 = ch(\gamma^{n+m}(x)) = \bar{c}_{n+m}(x)$. Due to the “gap” in the cohomology of X , we find that, for $k > n$, we have

$$ch(\gamma^k(x)) = 0.$$

By the particular cohomological properties of X , the Chern character is injective for elements of filtration $> n$ in $\tilde{K}(X)$ (see [AtHi]). Being zero or of filtration $\geq k$ (as Proposition 2.2 shows), $\gamma^k(x)$ has to vanish for $k > n$. This concludes the proof. \square

11. A “DOUBLING FORMULA” FOR STIRLING NUMBERS OF THE SECOND KIND

In the present section, we calculate the γ -operations for the product $S^{2n} \times S^{2m}$. From this computation and Proposition 10.1, we deduce again the γ -cone, as appearing in Theorem 7.1. This example illustrates that computing the c -cone is in general easier than computing the γ -cone. On the other hand, the latter calculation leads to an interesting “doubling formula” for Stirling numbers of the second kind. We will also conjecture the analogous formula for Stirling numbers of the first kind.