

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 47 (2001)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE POSITIVE CONE OF SPHERES AND SOME PRODUCTS OF SPHERES
Autor: MATTHEY, Michel / SUTER, Ulrich
Kurzfassung
DOI: <https://doi.org/10.5169/seals-65432>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 18.05.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

THE POSITIVE CONE OF SPHERES AND SOME PRODUCTS OF SPHERES

by Michel MATTHEY *) and Ulrich SUTER

ABSTRACT. Motivated by Elliott's K -theoretic classification of C^* -algebras of type AF, we compute the positive cone of the K -theory of some spaces. These include the spheres, the products of an odd-dimensional sphere by a sphere, the products of the 2-sphere by a sphere, and of the products $S^4 \times S^4$, $S^4 \times S^6$, $S^6 \times S^6$ and $S^6 \times S^8$. This amounts to computing the geometric dimension of stable classes of complex vector bundles over these spaces. We establish a few general properties of the positive cone and of approximations to it, the γ -cone and the c -cone. We also get information on the Whitehead product structure in the homotopy groups of $BU(n)$. Moreover, we prove a “doubling formula” for Stirling numbers of the second kind.

1. INTRODUCTION

Let $\mathcal{G}(S)$ be the Grothendieck group completion of an abelian semigroup S , and let $\theta: S \longrightarrow \mathcal{G}(S)$ be the corresponding universal homomorphism. The image of θ , denoted by $\mathcal{G}_+(S)$, is a sub-semigroup of $\mathcal{G}(S)$. If S has a zero, in other words if it is an abelian monoid, then $\mathcal{G}_+(S)$ induces a translation invariant preordering on $\mathcal{G}(S)$ (i.e. a reflexive and transitive relation, but not necessarily antisymmetric). The elements of $\mathcal{G}_+(S)$ are called *positive* and $\mathcal{G}_+(S)$ is called the *positive cone* (see [Ell] and [Bla1]). The pair $(\mathcal{G}(S), \mathcal{G}_+(S))$ is an isomorphism invariant of S , and a basic question is: to what extent does this invariant characterize the abelian semigroup S ?

The above notions are of interest in connection with the classification problem of C^* -algebras. For a unital C^* -algebra A , let $S = \text{Proj}(A)$ be the abelian monoid of equivalence classes of projectors in the matrix algebra $\mathbf{M}_\infty(A)$. The K -theory of A , denoted by $K_0(A)$ or $K(A)$, is by definition

*) Partially supported by the Swiss National Science Foundation grant 20-56816.99