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Autor: BUSCH, Cornelia

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representation $\rho_{\mathcal{I}}\colon \mathbf{Z}/p\mathbf{Z} \to \mathrm{Sp}(p-1,\mathbf{Z})$ is the one corresponding to $\widetilde{\rho}_{\mathcal{I}}$. We have an induced homomorphism

$$\rho_{\mathcal{I}}^{*}\colon \mathrm{H}^{2j}(\mathrm{Sp}(p-1,\mathbf{Z}),\mathbf{Z}) \longrightarrow \mathrm{H}^{2j}(\mathbf{Z}/p\mathbf{Z},\mathbf{Z})$$
$$d_j(\mathbf{Z}) \longmapsto d_j(\rho_{\mathcal{I}}).$$

Herewith for any p the class $d_j(\mathbf{Z}) \in \mathrm{H}^{2j}(\mathrm{Sp}(p-1,\mathbf{Z}),\mathbf{Z})$ is nonzero and has either infinite order or finite order divisible by p, since it restricts non-trivially to $\mathrm{H}^{2j}(\mathbf{Z}/p\mathbf{Z},\mathbf{Z})$. This shows that $d_j(\mathbf{Z}) \in \mathrm{H}^{2j}(\mathrm{Sp}(\mathbf{Z}),\mathbf{Z})$ has infinite order. □

This is a new proof of a result of A. Borel [3]. He proved that $\mathrm{H}^*(\mathrm{Sp}(\mathbf{Z}),\mathbf{Q}) = \mathbf{Q}[d_1, d_3, \dots]$. Moreover, each d_{2i} can be expressed as a polynomial in the d_{2j+1}'s. This implies that all the $d_i(\mathbf{Z})$'s have infinite order.

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Cornelia Busch

Departement Mathematik
ETH Zürich
CH-8092 Zürich
Switzerland
email: cbusch@math.ethz.ch