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representation $\rho_{\mathcal{I}}: \mathbf{Z}/p\mathbf{Z} \rightarrow \mathrm{Sp}(p-1, \mathbf{Z})$ is the one corresponding to $\tilde{\rho}_{\mathcal{I}}$. We have an induced homomorphism

$$\begin{aligned}\rho_{\mathcal{I}}^*: H^{2j}(\mathrm{Sp}(p-1, \mathbf{Z}), \mathbf{Z}) &\longrightarrow H^{2j}(\mathbf{Z}/p\mathbf{Z}, \mathbf{Z}) \\ d_j(\mathbf{Z}) &\longmapsto d_j(\rho_{\mathcal{I}}).\end{aligned}$$

Herewith for any p the class $d_j(\mathbf{Z}) \in H^{2j}(\mathrm{Sp}(p-1, \mathbf{Z}), \mathbf{Z})$ is nonzero and has either infinite order or finite order divisible by p , since it restricts nontrivially to $H^{2j}(\mathbf{Z}/p\mathbf{Z}, \mathbf{Z})$. This shows that $d_j(\mathbf{Z}) \in H^{2j}(\mathrm{Sp}(\mathbf{Z}), \mathbf{Z})$ has infinite order. \square

This is a new proof of a result of A. Borel [3]. He proved that $H^*(\mathrm{Sp}(\mathbf{Z}), \mathbf{Q}) = \mathbf{Q}[d_1, d_3, \dots]$. Moreover, each d_{2i} can be expressed as a polynomial in the d_{2j+1} 's. This implies that all the $d_i(\mathbf{Z})$'s have infinite order.

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