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flow is not the geodesic flow (up to a finite cover), the action of  $\Gamma$  on  $S^1_u$  is faithful. For instance, we get in this way some examples of faithful actions of the fundamental group of some hyperbolic 3-manifolds on the circle.

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