

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 47 (2001)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: GROUPS ACTING ON THE CIRCLE
Autor: GHYS, Étienne
Kapitel: 1. Introduction
DOI: <https://doi.org/10.5169/seals-65441>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 17.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

GROUPS ACTING ON THE CIRCLE

by Étienne GHYS

1. INTRODUCTION

The classical theory of dynamical systems studies the orbit structure of a homeomorphism or of a flow on a manifold, *i.e.* of actions of the group \mathbf{Z} or \mathbf{R} . This theory can be generalized to actions of an arbitrary group Γ on a manifold. These notes propose a survey of some results concerning the case where the group Γ is quite arbitrary and the manifold is the circle or the real line.

This paper covers a very small part of the theory. We decided to discuss only the topological aspect; this is a pity since the theory of groups of smooth diffeomorphisms is so rich! For instance, we would have liked to discuss the so called “level theory” around Sacksteder’s theorem or problems related to structural stability. Even in the restricted domain of topological dynamics, these notes are incomplete; we should have discussed at least the remarkable classification of convergence groups due to Tukia, Casson-Jungreis, Gabai [15, 24]... The author hopes that in the near future he will be able to write a reasonably complete survey on this area.

Our main goal is to provide a motivation for our paper on actions of higher rank lattices on the circle [26]. Section 3 describes some important examples of group actions on the circle. Section 4 reviews some of the main topological and algebraic properties of the group of homeomorphisms of the circle. In Sections 5 and 6 we describe the interplay between the classical rotation number and the cohomological invariant given by the Euler class. Finally, in Section 7 we discuss recent results concerning actions of lattices on the circle. Subsection 7.2 is essentially an extract from [26].

A first version of these notes was prepared for the 13th ELAM in Lima, in June 1999.

I would like to thank John Crisp, Bruno Sévenec and Dave Witte for interesting comments on this text.