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## GROUPS ACTING ON THE CIRCLE

by Étienne GHYS

### 1. INTRODUCTION

The classical theory of dynamical systems studies the orbit structure of a homeomorphism or of a flow on a manifold, *i.e.* of actions of the group **Z** or **R**. This theory can be generalized to actions of an arbitrary group  $\Gamma$  on a manifold. These notes propose a survey of some results concerning the case where the group  $\Gamma$  is quite arbitrary and the manifold is the circle or the real line.

This paper covers a very small part of the theory. We decided to discuss only the topological aspect; this is a pity since the theory of groups of smooth diffeomorphisms is so rich! For instance, we would have liked to discuss the so called “level theory” around Sacksteder’s theorem or problems related to structural stability. Even in the restricted domain of topological dynamics, these notes are incomplete; we should have discussed at least the remarkable classification of convergence groups due to Tukia, Casson-Jungreis, Gabai [15, 24]... The author hopes that in the near future he will be able to write a reasonably complete survey on this area.

Our main goal is to provide a motivation for our paper on actions of higher rank lattices on the circle [26]. Section 3 describes some important examples of group actions on the circle. Section 4 reviews some of the main topological and algebraic properties of the group of homeomorphisms of the circle. In Sections 5 and 6 we describe the interplay between the classical rotation number and the cohomological invariant given by the Euler class. Finally, in Section 7 we discuss recent results concerning actions of lattices on the circle. Subsection 7.2 is essentially an extract from [26].

A first version of these notes was prepared for the 13<sup>th</sup> ELAM in Lima, in June 1999.

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