

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 47 (2001)  
**Heft:** 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** FINITE TYPE LINK-HOMOTOPY INVARIANTS  
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**Kurzfassung**  
**DOI:** <https://doi.org/10.5169/seals-65440>

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## FINITE TYPE LINK-HOMOTOPY INVARIANTS

by Xiao-Song LIN<sup>\*</sup>)

ABSTRACT. An explicit polynomial in the linking numbers  $l_{ij}$  and Milnor's triple linking numbers  $\mu(rst)$  on six component links is shown to be a well-defined finite type link-homotopy invariant. This solves a problem raised by B. Mellor and D. Thurston. An extension of our construction also produces a finite type link invariant which detects the invertibility for some links.

### 1. INTRODUCTION

The classification of links in 3-space up to link-homotopy [3] was published ten years ago. Since then, the question of whether one could extract link-homotopy invariants from this classification has not been addressed properly. Recall that this classification starts with the classification of  $k$  component string links up to link-homotopy by a finitely generated torsion free nilpotent group  $\mathcal{H}(k)$ . Then link-homotopy classes are classified as orbits of this group  $\mathcal{H}(k)$  under the “nilpotent action” of conjugations and partial conjugations. The group  $\mathcal{H}(k)$  is of rank

$$\sum_{n=2}^k (n-2)! \binom{k}{n},$$

so an element of  $\mathcal{H}(k)$  can be described uniquely by that many integers.

These integers are Milnor's  $\mu$ -numbers<sup>1)</sup> with distinct indices. By a *link-homotopy invariant polynomial*, or simply a link-homotopy invariant, we mean a polynomial in these  $\mu$ -numbers which is invariant under the action of

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<sup>\*</sup>) Partially supported by the Overseas Youth Cooperation Research Fund of NSFC and a grant from NSF.

<sup>1)</sup> Usually, they are called  $\mu$ -invariants. But the word “invariant” is clearly abused here, so we decide to call them  $\mu$ -numbers.