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FINITE TYPE LINK-HOMOTOPY INVARIANTS

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ABSTRACT. An explicit polynomial in the linking numbers l_{ij} and Milnor's triple linking numbers $\mu(rst)$ on six component links is shown to be a well-defined finite type link-homotopy invariant. This solves a problem raised by B. Mellor and D. Thurston. An extension of our construction also produces a finite type link invariant which detects the invertibility for some links.

1. INTRODUCTION

The classification of links in 3-space up to link-homotopy [3] was published ten years ago. Since then, the question of whether one could extract link-homotopy invariants from this classification has not been addressed properly. Recall that this classification starts with the classification of k component string links up to link-homotopy by a finitely generated torsion free nilpotent group $\mathcal{H}(k)$. Then link-homotopy classes are classified as orbits of this group $\mathcal{H}(k)$ under the "nilpotent action" of conjugations and partial conjugations. The group $\mathcal{H}(k)$ is of rank

$$\sum_{n=2}^{k}(n-2)!\binom{k}{n},$$

so an element of $\mathcal{H}(k)$ can be described uniquely by that many integers.

These integers are Milnor's μ-numbers[1]) with distinct indices. By a *link-homotopy invariant polynomial*, or simply a link-homotopy invariant, we mean a polynomial in these μ-numbers which is invariant under the action of

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[1]) Usually, they are called μ-invariants. But the word "invariant" is clearly abused here, so we decide to call them μ-numbers.