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$\operatorname{Re}(1/\kappa) + m > -1$  and bounded away from  $-1$  as  $\kappa \rightarrow -1/p$ . For the  $\gamma_\kappa$  we can take *circular arcs* lying in the fourth quadrant, *orthogonal to the real axis* and running from  $0$  to  $(1 - \kappa)/(1 + \kappa)$ ; these  $\gamma_\kappa$  will stay away from  $1$  while  $(1 - \kappa)/(1 + \kappa) \rightarrow (p + 1)/(p - 1)$ , i.e., while  $\kappa \rightarrow -1/p$ . Then  $|1 - s|$  will be uniformly bounded away from  $0$  for  $s$  on these  $\gamma_\kappa$ , and the *integral* in (102) thus *remain bounded* as  $\kappa \rightarrow -1/p$ . At the same time, however, the *sum* in (102) will tend to  $\infty$  like  $1/((1/\kappa) + p)$ . Hence  $A(\kappa)$  will tend to  $\infty$  as  $\kappa \rightarrow -1/p$  in the manner described.

This being so for *each* of the points  $-1/p$ ,  $p = 2, 3, \dots$ ,  $A(\kappa)$  can have *no analytic continuation from  $\mathcal{D}_+$  into any neighbourhood of  $0$* , since any such continuation would have to coincide with the one just constructed on the intersection of the open second quadrant with the neighbourhood in question. That is what we needed to prove.

Before concluding, I must again thank my friend Victor Havin for having, during a conversation, expressed a thought which, indirectly, got me onto a path leading to the above argument.

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