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$\lambda^{(1)} \supset \lambda^{(2)} \supset \dots \supset \lambda^{(n)} \supset \emptyset$ , and from these partitions one successively fills in the entries in the northwest to southeast diagonal rows of the hive; the rhombus inequalities (1)–(3) are automatically satisfied.

To make the story complete, we recall why such contratableaux correspond to Littlewood-Richardson skew tableaux, using standard results about tableaux, as in [5]. However, it may be pointed out that these contratableaux are at least as easy to produce and enumerate as the more classical skew tableaux. First, the condition that  $w(T) \cdot w(U(\mu))$  is a reverse lattice word, given that the number of times  $i$  occurs in  $T$  is  $\nu_i - \mu_i$ , is equivalent to asserting that  $w(T) \cdot w(U(\mu))$  is Knuth equivalent to  $w(U(\nu))$  [5, §5.2]. The rectification  $R$  of a contratableau  $T$  of shape  $\lambda$  is easily seen to be a tableau of shape  $\lambda$ , and with the same property that  $w(R) \cdot w(U(\mu))$  is Knuth equivalent to  $w(U(\nu))$ . The correspondence between tableaux and contratableaux of shape  $\lambda$  is a bijection, by reversing the rectification process.

Now the condition that  $w(R) \cdot w(U(\mu))$  be Knuth equivalent to  $w(U(\nu))$  is equivalent to the condition that  $R \cdot U(\mu) = U(\nu)$  in the plactic monoid of tableaux [5, §2.1]. It is easy to see, from the definition of multiplying tableaux by column bumping entries of the first tableau into the second [5, §A.2], that if  $R$  and  $S$  are tableaux with  $R \cdot S = U(\beta)$ , then  $S$  must be equal to  $U(\alpha)$  for some partition  $\alpha$ . This gives a correspondence between the set of tableaux  $R$  that we are looking at and the set of pairs  $(R, S)$  with  $R$  of shape  $\lambda$ ,  $S$  of shape  $\mu$ , whose product is the tableau  $U(\nu)$ . There is a standard construction [5, §5.1] between these pairs and the set of skew tableau on the shape  $\nu/\lambda$  of content  $\mu$  whose word is a reverse-lattice word.

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