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The last comment in the proof above and our earlier remarks about actions on S^{2^t-1} also imply the following: If π acts freely and topologically on S^{2^t-1} , then π satisfies all pq -conditions. On S^{2^s-1} with $2s \neq 2^t$ there are always free smooth actions by metacyclic groups $D_{p,q}$ violating the pq -conditions.

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