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via the “asymptotic stretch” of  $f_*: \pi_1 \rightarrow \pi_1$  with respect to the word metric in  $\pi_1$  (see [Bow]). But this is not sharp even for linear automorphisms of tori  $T^n$ , where the entropy is expressed by the “ $k$ -dimensional stretch” on  $H_1$  for some  $k \leq n$  that equals the spectral radius of  $f_*$  on  $H_k$ . Such  $k$ -stretch can be defined, in general, in terms of  $f_*: \pi_1(S) \rightarrow \pi_1(S)$  and the classifying map  $S \rightarrow K(\pi_1, 1)$  (refining the spectral radius of  $f_*$  on  $H^k$  coming from  $K(\pi_1, 1)$ ), but my obvious “proof” of the lower bound on the entropy by this  $k$ -stretch missed a hidden trap. This was also overlooked in [Ma<sub>3</sub>] (for  $f_*: H_1 \rightarrow H_1$ , where a proper identification of the “ $k$ -stretch” with the spectral radius needs extra work), as was pointed out to me much later by David Fried. (The difficulty already appears for closed subsets  $S$  in the torus  $T^n$  invariant under linear automorphisms  $f$  of  $T^n$ , where one wishes to estimate the entropy of  $f|_S$  in terms of  $f_*$  acting on the *spectral* cohomology of  $S$  coming from  $T^n$ . On the other hand, the case of  $T^n \rightarrow T^n$  is settled in [Mi-Pr].)

APPENDIX 6. Probably, the recent progress in Nielsen theory allows a description of the cases, where  $\text{card}(\text{Fix} f)$  is well controlled from below by some twisted Lefschetz number (see [Fel]).

APPENDIX 7. Nothing interesting to say.

APPENDIX 8. Minima of geometric functionals related to the logical complexity have been studied in depth by A. Nabutovski (see [Na] and references therein). Yet I do not feel ready yet to write this Appendix. For example, I do not see what is the actual influence of a suitable (?) complexity measure of  $\pi_1(V)$  on the Plateau problem in  $V$ .

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