Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 46 (2000)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE WITT GROUP OF LAURENT POLYNOMIALS

Autor: Ojanguren, Manuel / Panin, Ivan

Kurzfassung

DOI: https://doi.org/10.5169/seals-64807

L'Enseignement Mathématique, t. 46 (2000), p. 361–383

THE WITT GROUP OF LAURENT POLYNOMIALS

by Manuel OJANGUREN and Ivan PANIN

ABSTRACT. We give a direct, self-contained proof of the fact that for a large class of rings A, in particular for all regular rings with involution, $W(A[t, 1/t]) = W(A) \oplus W(A)$.

1. INTRODUCTION

The purpose of this note is to give a short direct proof of two fundamental theorems on the Witt group of polynomials and Laurent extensions of a ring A. These theorems were proved independently by M. Karoubi [3] and by A. Ranicki [5]. We will state them under the most general conditions on A and for their proofs we will use nothing more than a general result on the K-theory of Laurent polynomials. In the last section we will show, by two counterexamples, that the assumptions we make on A are necessary.

We begin by recalling briefly some definitions. We refer to [4] for a more detailed exposition and for the proofs of the few basic results that we will use.

Let A be an associative ring with an involution denoted by $a \mapsto a^\circ$. Except in §2 we will always assume that 2 is invertible in A. If M is a right A-module, we denote by M^* its dual $\mathrm{Hom}_A(M,A)$ endowed with the right action of A given by $fa(x) = a^\circ f(x)$ for any $f\colon M \to A$ and $a \in A$. If P is a finitely generated projective right A-module we identify it with P^{**} through the canonical isomorphism mapping $x \in P$ to $\widehat{x}\colon P^* \to A$ defined by $\widehat{x}(f) = f(x)$.

Let ϵ be 1 or -1. An *ϵ-hermitian space* over A is a pair (P, α) consisting of a finitely generated projective right A-module P and an A-isomorphism $\alpha\colon P \to P^*$ satisfying $\alpha = \epsilon\alpha^*$. For brevity ϵ-hermitian spaces will be called *spaces*. A 1-hermitian space (over a commutative ring A) is also called a *quadratic space.*