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who has not can forget about this part of the theorem". It is hard to decide whether these assertions about a *general reader* are correct. There is little evidence of the second claim. For the first there is more evidence. Indeed, it is true that, in order to reduce to the case when the ring is an integral domain, it is not hard to see that it suffices to prove the following assertion: Let M be the generic matrix over $\mathbf{Q}$, and let L be the splitting field, over $\mathbf{Q}(M)$, of the characteristic polynomial of M. Then L has degree $n!$ over $\mathbf{Q}(M)$. The latter assertion follows, using standard methods, from the theory of integral Galois extensions (see e.g. [L], Chapter VII, §2, Proposition 2.5, p. 342). Apparently the methods are foreign to the problem, and the results on integral ring extensions that are used are more difficult than the result that we want to prove. In this article we follow a more natural path, resulting in a simple, self contained, and short proof of the Spectral Mapping Theorem. As a consequence we get a better understanding of the result and we can place it into a more general framework.

The proof suggests that the Spectral Mapping Theorem should be considered within the framework of norms on algebras. Our method leads to a uniqueness result for norms on the polynomial ring in one variable from which a generalized Spectral Mapping Theorem follows. Applied to the most common norms on the polynomial ring in one variable the uniqueness gives generalizations of classical formulas for the resultant of polynomials.

2. The Spectral Mapping Theorem

Let M be an $n \times n$-matrix with entries in a commutative ring k with unity. Assume that the characteristic polynomial $P_M(t) = \det(tI_n - M)$ of M splits completely in k, that is $P_M(t) = \prod_{i=1}^n (t - \lambda_i)$ with $\lambda_i \in k$ for $i = 1, \dots, n$.

The *Spectral Mapping Theorem* states that, for every polynomial $F(x)$ in the variable x with coefficients in an arbitrary commutative ring R that contains k as a subring, we have

$$\det F(M) = \prod_{i=1}^{n} F(\lambda_i) \tag{2.1}$$

in R. In particular, when $f(x)$ is a polynomial with coefficients in k and we use (2.1) for the ring $k[t]$ and the polynomial $F(x) = t - f(x)$, we obtain in $k[t]$:

$$P_{f(M)}(t) = \det(tI_n - f(M)) = \det F(M) = \prod_{i=1}^{n} F(\lambda_i) = \prod_{i=1}^{n} (t - f(\lambda_i)) .$$