THE SPECTRAL MAPPING THEOREM, NORMS ON RINGS, AND RESULTANTS

L'Enseignement Mathématique, t. 46 (2000), p. 349–358

THE SPECTRAL MAPPING THEOREM, NORMS ON RINGS, AND RESULTANTS

by D. LAKSOV, L. SVENSSON and A. THORUP

ABSTRACT. We give a short, simple and self-contained proof of the Spectral Mapping Theorem for matrices with entries in an arbitrary commutative ring. The result is placed in the wider framework of norms on algebras. It is shown that the Spectral Mapping Theorem follows from a uniqueness result for norms on polynomial rings in one variable. The results are used to generalize classical formulas for the resultant of polynomials.

1. INTRODUCTION

It is a well-known and useful result in spectral theory of complex finite dimensional vector spaces that if the characteristic polynomial of an $n \times n$-matrix M splits as $P_M(t) = \det(tI_n - M) = \prod_{i=1}^n (t - \lambda_i)$ then, for any polynomial $F(x)$, we have that $\det(tI_n - F(M)) = \prod_{i=1}^n (t - F(\lambda_i))$. We call this result the Spectral Mapping Theorem, because it is similar to the Spectral Mapping Theorem for Banach algebras. Many proofs of the result for complex finite dimensional vector spaces are known, most of them based upon transforming the matrix into triangular form (see [B2], §5, Proposition 10, p. 36), or using the Jordan canonical form for the matrix (see [L], Chapter XIV, §3, Theorem 3.10, p. 566). The Theorem and its proofs are easily generalized to arbitrary fields, and therefore to integral domains. In our work on parameter spaces in algebraic geometry ([L-S], [S1], [S2]) we needed a generalization of the Spectral Mapping Theorem to matrices with entries in arbitrary commutative rings with unity. The only reference we could find to such a generalization was [L], Chapter XIV, §3, Theorem 3.10, p. 566, where a proof is deduced from the theory of integral ring extensions. The difficult part of the proof is dismissed with the phrase "This is obvious to the reader who read the chapter on integral ring extensions, and the reader who has not can forget about this part of the theorem". It is hard to decide whether these assertions about a *general reader* are correct. There is little evidence of the second claim. For the first there is more evidence. Indeed, it is true that, in order to reduce to the case when the ring is an integral domain, it is not hard to see that it suffices to prove the following assertion: Let M be the generic matrix over $\mathbf{Q}$, and let L be the splitting field, over $\mathbf{Q}(M)$, of the characteristic polynomial of M. Then L has degree $n!$ over $\mathbf{Q}(M)$. The latter assertion follows, using standard methods, from the theory of integral Galois extensions (see e.g. [L], Chapter VII, §2, Proposition 2.5, p. 342). Apparently the methods are foreign to the problem, and the results on integral ring extensions that are used are more difficult than the result that we want to prove. In this article we follow a more natural path, resulting in a simple, self contained, and short proof of the Spectral Mapping Theorem. As a consequence we get a better understanding of the result and we can place it into a more general framework.

The proof suggests that the Spectral Mapping Theorem should be considered within the framework of norms on algebras. Our method leads to a uniqueness result for norms on the polynomial ring in one variable from which a generalized Spectral Mapping Theorem follows. Applied to the most common norms on the polynomial ring in one variable the uniqueness gives generalizations of classical formulas for the resultant of polynomials.

2. The Spectral Mapping Theorem

Let M be an $n \times n$-matrix with entries in a commutative ring k with unity. Assume that the characteristic polynomial $P_M(t) = \det(tI_n - M)$ of M splits completely in k, that is $P_M(t) = \prod_{i=1}^{n}(t - \lambda_i)$ with $\lambda_i \in k$ for $i = 1, \dots, n$.

The *Spectral Mapping Theorem* states that, for every polynomial $F(x)$ in the variable x with coefficients in an arbitrary commutative ring R that contains k as a subring, we have

$$\det F(M) = \prod_{i=1}^{n} F(\lambda_i) \tag{2.1}$$

in R. In particular, when $f(x)$ is a polynomial with coefficients in k and we use (2.1) for the ring $k[t]$ and the polynomial $F(x) = t - f(x)$, we obtain in $k[t]$:

$$P_{f(M)}(t) = \det(tI_n - f(M)) = \det F(M) = \prod_{i=1}^{n} F(\lambda_i) = \prod_{i=1}^{n}(t - f(\lambda_i)).$$

3. A REDUCTION

To prove the Spectral Mapping Theorem it suffices to verify that it holds for the polynomial ring $k[t_0, \dots, t_m]$ in variables $t_0, \dots, t_m$ over k, and for the polynomial $F(x) = t_0 + t_1 x + \cdots + t_m x^m$. This is because any polynomial $G(x) = b_0 + b_1 x + \cdots + b_m x^m$ with coefficients in a ring R containing k as a subring is the image of $F(x)$ by the map $g\colon k[t_0, \dots, t_m][x] \to R[x]$ defined by $g(a) = a$ for $a \in k$, by $g(t_i) = b_i$ for $i = 0, \dots, m$, and by $g(x) = x$. If we can prove the equality $\det F(M) = \prod_{i=1}^n F(\lambda_i)$ in $k[t_0, \dots, t_m]$ we obtain that $\det G(M) = g(\det F(M)) = \prod_{i=1}^n g(F(\lambda_i)) = \prod_{i=1}^n G(\lambda_i)$ in R.

4. THE PROOF

Clearly (2.1) holds when F is a constant a where it simply states that $\det(aI_n) = a^n$. We shall prove (2.1) for polynomials F of degree $m > 0$ by induction on m.

We first note that if $F(x)$ has a root λ in R, so that $F(x) = (x - \lambda)G(x)$ in $R[x]$, then (2.1) holds for $F(x)$. Indeed, $G(x)$ is of degree $m - 1$ so it follows from the induction hypothesis that $\det G(M) = \prod_{i=1}^n G(\lambda_i)$. Since $F(M) = (M - \lambda I_n)G(M)$ we obtain:

$$\begin{aligned}\det F(M) &= \det(M - \lambda I_n) \det G(M) \\ &= \prod_{i=1}^n (\lambda_i - \lambda) \prod_{i=1}^n G(\lambda_i) = \prod_{i=1}^n (\lambda_i - \lambda) G(\lambda_i) = \prod_{i=1}^n F(\lambda_i).\end{aligned}$$

As we saw in Section 3 it suffices to prove the result for the ring $Q = k[t_0, \dots, t_m]$ and the polynomial $F(x) = t_0 + t_1 x + \cdots + t_m x^m$. Let x and y be independent variables over the ring Q. The polynomial $F(x) - F(y)$ in x with coefficients in $Q[y]$ has the root $x = y$. Hence, as we just observed, (2.1) holds for the polynomial $F(x) - F(y)$. We obtain the equation:

$$\det(F(M) - F(y)I_n) = \prod_{i=1}^n (F(\lambda_i) - F(y)) \tag{4.1}$$

in $Q[y]$.

The equation (2.1) is a consequence of (4.1). To see this we observe that $F(y)$ in $Q[y]$ is transcendent over Q, that is the element $F(y)$ in $Q[y]$ does not satisfy a polynomial relation $a_0 + a_1 F(y) + \cdots + a_l F(y)^l = 0$ with coefficients a_i in Q and $a_l \neq 0$, because the coefficient $a_l t_m^l$ of the highest power y^{ml} of y that appears in the relation is non-zero. It follows that we can define a homomorphism of rings $h\colon Q[F(y)] \to Q$ by $h(a) = a$ for $a \in Q$, and $h(F(y)) = 0$. We apply the map h to both sides of (4.1) and obtain the equality (2.1).

5. Norms on algebras

The only properties of determinants that we used in the proof of the Spectral Mapping Theorem is that they are multiplicative, functorial and homogeneous. It is therefore natural to place the proof into the more general framework of norms on algebras. The advantage of this point of view is that we obtain a deeper understanding of the Spectral Mapping Theorem, and we obtain a natural connection with resultants of polynomials.

A *norm* N of *degree* n on a, not necessarily commutative, k-algebra A is a family of maps $N_R\colon R \otimes_k A \to R$, one for every commutative k-algebra R, that satisfies the conditions:

(1) $N_R(a \otimes 1) = a^n$ for all elements a in R.

(2) $N_R(uv) = N_R(u)N_R(v)$ for all elements u and v of $R \otimes_k A$.

(3) For every homomorphism $\varphi\colon R \to S$ of commutative k-algebras we have $\varphi N_R = N_S(\varphi \otimes \mathrm{id}_A)$.

A norm on an algebra may be described as a *multiplicative homogeneous polynomial law* (see Roby [R], or [B1], §9, Définition 3, p. 52).

For any map $B \to A$ of k-algebras the norm N on A *restricts* to a norm on B of degree n. Moreover, for every homomorphism of commutative rings $k \to k'$ the norm N on A *induces* a norm of degree n on the k'-algebra $k' \otimes_k A$.

Let N be a norm of degree n on a k-algebra A. Denote by $k[t]$ the k-algebra of polynomials in the variable t with coefficients in k. For every element α in A the polynomial in $k[t]$:

$$P_\alpha(t) = P_\alpha^N(t) = N_{k[t]}(t - \alpha)$$

is called the *characteristic polynomial* of α. The *trace* $\mathrm{Tr}^N(\alpha)$ of α is the element in k such that $-\mathrm{Tr}^N(\alpha)$ is the coefficient of t^{n-1} in $P_\alpha(t)$.

We note that $P_\alpha(0) = (-1)^n N_k(\alpha)$.

5.1. LEMMA. *Let N be a norm of degree n on a k-algebra A. For each element α of A the characteristic polynomial $P_\alpha^N(t) = N_{k[t]}(t-\alpha)$ is monic of degree n.*

Moreover, the trace Tr^N *is a k-linear map $A \to k$.*

Proof. Let s, t, u, v be independent variables over the ring k. For each element β in A the norm $N_{k[s,t,u]}(t - \alpha s - \beta u)$ is a polynomial in $k[s,t,u]$. Since N is of degree n we have that $N_{k[s,t,u,v]}(vt - \alpha vs - \beta vu) = v^n N_{k[,s,t,u]}(t-\alpha s-\beta u)$. It follows that $N_{k[s,t,u]}(t-\alpha s-\beta u)$ is homogeneous of degree n in $k[s,t,u]$. In particular the coefficient of t^{n-1} is of the form $as+bu$ with a and b in k. By evaluating the polynomial $N_{k[s,t,u]}(t-\alpha s-\beta u)$ at $s=0$, $u=0$, it follows that the coefficient to t^n is equal to 1. Hence $N_{k[t]}(t-\alpha)$ is a monic polynomial of degree n, and $a = -\mathrm{Tr}^N(\alpha)$. Similarly, $b = -\mathrm{Tr}^N(\beta)$. Hence we have that $\mathrm{Tr}^N(\alpha s + \beta u) = -(as+bu) = \mathrm{Tr}^N(\alpha)s + \mathrm{Tr}^N(\beta)t$. Specializing s and t to any pair of elements of k the second part of the Lemma follows. □

5.2. EXAMPLE. Let M be a free module of rank n over k, or more generally a projective k-module of constant rank n. Then the determinant defines a norm of degree n on $\mathrm{End}_k(M)$.

Let A be a k-algebra which is free of rank n as a k-module. Left multiplication by elements of A define an injection $A \to \mathrm{End}_k(A)$ of k-algebras. By restriction we obtain a norm of degree n on A.

6. NORMS AND RESULTANTS

Let $F(x) = f_0 + \cdots + f_m x^m$ and $P(x) = p_0 + \cdots + p_n x^n$ be polynomials of degree m, respectively n in the k-algebra $k[x]$ of polynomials in the variable x with coefficients in k. The *resultant* $\mathrm{Res}(F,P)$ of F and P is the determinant of the $(m+n)\times(m+n)$-matrix $D(F,P)$ whose columns are the coefficients of the polynomials $F, xF, \dots, x^{n-1}F, P, xP, \dots, x^{m-1}P$. Note that the definition is asymmetric in F and P in the sense that $\mathrm{Res}(F,P) = (-1)^{mn}\mathrm{Res}(P,F)$.

When P is monic the resultant is equal to the determinant of the endomorphism induced by multiplication by F on the free k-module $k[x]/(P(x))$ of rank n. To see this we note that for $i = 0, \dots, n-1$ we can write $x^i F = Q_i P + R_i$ in $k[x]$, where $Q_i(x)$ and $R_i(x)$ are of degrees at most $m-1$, respectively $n-1$. It follows that the determinant of $D(F,P)$ is equal to the determinant of the $(m+n)\times(m+n)$-matrix $B(F,P)$ whose columns are the coefficients of the polynomials $R_0, \dots, R_{n-1}, P, xP, \dots, x^{m-1}P$. We see that the $n \times n$-block $C(F,P)$ in the upper left corner of $B(F,P)$ is the matrix $C(F,P)$ of the map induced by multiplication by F on $k[x]/(P(x))$, and the $m \times m$-block in the lower right corner is upper triangular with 1's on the diagonal. Moreover, the entries of $C(F,P)$ are the only non-zero entries in the first n columns of $B(F,P)$. It follows that $\operatorname{Res}(F,P) = \det C(F,P)$, as we claimed.

6.1. EXAMPLE. When P is a monic polynomial we saw in Example 5.2 that the k-algebra $k[x]/(P(x))$ which is free of rank n as a k-module has a canonical norm. Via the canonical map $k[x] \to k[x]/(P(x))$ we obtain a canonical norm N'_P on $k[x]$. The above interpretation of the resultant can then be written as

$$(N'_P)_R(F) = \operatorname{Res}(F,P) \tag{6.1.1}$$

for all commutative k-algebras R and all polynomials $F(x)$ in $R[x] = R \otimes_k k[x]$. By an easy computation of the determinant defining $\operatorname{Res}(t-x,P)$, we obtain that the characteristic polynomial of x with respect to N'_P is

$$P_x^{N'_P}(t) = P(t)\,.$$

6.2. EXAMPLE. We shall introduce a second important norm on $k[x]$. Let $P(x)$ be a monic polynomial of degree n in the k-algebra $k[x]$. There is a canonical ring extension $k \subseteq k' = k[\lambda_1, \dots, \lambda_n]$ such that $P(x)$ splits as $P(x) = \prod_{i=1}^n (x - \lambda_i)$ in $k'[x]$. The extension is obtained by induction starting with $k = k_0$ and $P_0(x) = P(x)$, and constructing $k_i = k[\lambda_1, \dots, \lambda_i]$ and $P_i(x) \in k[\lambda_1, \dots, \lambda_i][x]$ from k_{i-1} and P_{i-1}, for $i = 1, 2, \dots, n$, by $k_i = k_{i-1}[x]/(P_{i-1}(x)) = k_{i-1}[\lambda_i]$, where λ_i is the class of x, and by $P_i(x) = P_{i-1}(x)/(x - \lambda_i)$. We note that k' is a free k-module of rank $n!$. The algebra k' is sometimes called the *universal decomposition algebra for* P (see [B1], §6, p. 68).

For every commutative k-algebra R and every polynomial G in $R[x] = R \otimes_k k[x]$ we have that $\prod_{i=1}^n G(\lambda_i)$ is symmetric in $\lambda_1, \dots, \lambda_n$, and consequently lies in the image of the inclusion $R \subseteq k' \otimes_k R$. We obtain a map $(N''_P)_R \colon R \otimes_k k[x] \to R$ defined by $(N''_P)_R(G) = \prod_{i=1}^n G(\lambda_i)$. In this way we obtain a norm N''_P of degree n on $k[x]$ and the characteristic polynomial of x with respect to the norm N''_P is

$$P_x^{N''_P}(t) = \prod_{i=1}^n (t - \lambda_i) = P(t)\,.$$

7. Uniqueness of norms and the Spectral Mapping Theorem

We are now ready to prove the uniqueness of norms on the polynomial ring $k[x]$ that we alluded to in the introduction. When we apply uniqueness to the norms N'_P and N''_P of Examples 6.1 and 6.2 we obtain the generalization of the Spectral Mapping Theorem for norms of rings also mentioned in the introduction. We also obtain some generalizations to rings of the classical interpretations of the resultant.

The proof of the uniqueness result is a slight variation of the proof of the Spectral Mapping Theorem given in Section 4.

The first formula in (7.2.1) below, when the norm is the determinant on the algebra of $n \times n$ matrices over an arbitrary ring k, was proved by McCoy [M], Theorem 56, p. 172.

7.1. Theorem. *A norm N on the k-algebra $k[x]$ of polynomials in the variable x is uniquely determined by the characteristic polynomial $P_x^N(t) = N_{k[t]}(t-x)$ of x with respect to N.*

Proof. Let N' be a second norm on $k[x]$ such that $P_x^N(t) = P_x^{N'}(t)$. Then N' and N are of the same degree n. The Theorem asserts that for any commutative k-algebra R and every polynomial F in $R[x] = R \otimes_k k[x]$ we have

$$N_R(F(x)) = N'_R(F(x)) \tag{7.1.1}$$

We prove (7.1.1) by induction on the degree m of $F(x)$. Clearly (7.1.1) holds when F is a constant. Assume that the degree m of F is positive and that the equality $N_R(G(x)) = N'_R(G(x))$ holds for all commutative k-algebras R and all polynomials $G(x)$ in $R[x]$ of degree $m-1$. Let t be an independent variable over $R[x]$. If the equality (7.1.1) holds for the polynomial $tx^m + F(x)$ in $R[t][x]$ it holds for $F(x)$, as we see by specializing t to 0. Consequently we may assume that the coefficient of x^m in $F(x)$ is a non-zero divisor in $R[x]$. Then the canonical map $R \to R[x]/(F(x))$ is an injection. Consequently we may also assume that R contains a root μ of $F(x)$. Then we have that $F(x) = F(x) - F(\mu) = (x-\mu)G(x)$ in $R[x]$ where $G(x)$ has degree $m-1$. Both sides of the equality (7.1.1) are multiplicative in $F(x)$, and the equality holds for $G(x)$ by the induction assumption. It also holds for $x - \mu$ as we see by specializing t to μ in the equality $N_{k[t]}(t-x) = P_x^N(t) = P_x^{N'}(t) = N'_{k[t]}(t-x)$. Hence we have proved the Theorem. $\square$

7.2. COROLLARY (The Generalized Spectral Mapping Theorem). *Let N be a norm of degree n on a k-algebra A, and let α be an element of A. For all polynomials F in $k[x]$ we have the equations*

$$N_k(F(\alpha)) = \operatorname{Res}(F, P_\alpha^N) = \prod_{i=1}^{n} F(\lambda_i) \tag{7.2.1}$$

in k, where $\lambda_1, \ldots, \lambda_n$ are elements of any extension $R \supseteq k$ such that $P_\alpha^N(t) = \prod_{i=1}^n (t - \lambda_i)$ in $R[x]$.

In particular we have that $\operatorname{Tr}^N(F(\alpha)) = \sum_{i=1}^n F(\lambda_i)$.

Proof. Let $P = P_\alpha^N$ be the characteristic polynomial of α with respect to N. The norm N restricts, via the canonical k-algebra homomorphism $k[x] \to A$ which sends x to α, to a norm on $k[x]$, and the characteristic polynomial of x with respect to this norm is P. On $k[x]$ we have the norm N, and the norms N'_P and N''_P of the Examples 6.1 and 6.2, and the characteristic polynomial of x with respect to all three norms is P. It follows from the Theorem that these three norms are equal. The equations (7.2.1) express the equality of the norms applied to the polynomial $F(x)$. Finally the expression for the trace follows by considering the coefficient of t^{n-1} of the left and right side of (7.2.1) applied to the polynomial $t - F(x)$ in $k[t][x]$. $\square$

The formula $\operatorname{Res}(F, P) = \prod_{i=1}^n F(\lambda_i)$ of Corollary (7.2) is the generalization to rings of the well-known interpretation of resultants by the roots of the monic polynomial P in the case when k is a field. If F is also monic and $F = \prod_{j=1}^m (x - \mu_j)$ in $R[x]$ we have

$$\operatorname{Res}(F, P) = \prod_{i=1}^{n} F(\lambda_i) = \prod_{i=1}^{n} \prod_{j=1}^{m} (\lambda_i - \mu_j)$$

which is often used as a definition of the resultant in the case k is an algebraically closed field.

8. THE DISCRIMINANT

We shall use the Generalized Spectral Mapping Theorem of Section 7 to prove two results on discriminants that are well-known for algebras of finite dimension over fields (see e.g. [B2], §5, Corollaire 6 and Corollaire 7, p. 38). Note that k below, as above, denotes a commutative ring with unity.

Let P be a monic polynomial in $k[x]$ of degree n. The *discriminant* of P is the element $(-1)^{n(n-1)/2}\operatorname{Res}(P',P)$ of k, where P' is the derivative of P.

Let $k \subseteq k' = k[\lambda_1,\dots,\lambda_n]$ be the canonical extension constructed in Example 6.2. We write $\Delta(P)=\prod_{i>j}(\lambda_i-\lambda_j)$. The formula for the resultant of Corollary 7.2 gives the equations

$$(-1)^{n(n-1)/2}\operatorname{Res}(P',P)=(-1)^{n(n-1)/2}\prod_{i\neq j}(\lambda_i-\lambda_j)=\prod_{i>j}(\lambda_i-\lambda_j)^2=\Delta(P)^2,$$

hence $\Delta(P)^2$ is the discriminant of P.

8.1. PROPOSITION. *Let N be a norm of degree n on a k-algebra A, and let α be an element of A. Denote by P_α the characteristic polynomial of α with respect to N. Then we have the equations in k:*

$$\det \operatorname{Tr}^N(\alpha^{p+q})_{p,q=0,\dots,n-1}=\Delta(P_\alpha)^2=(-1)^{n(n-1)/2}N_k(P'_\alpha(\alpha)).$$

Proof. By (7.2.1) we have the equation $N_k(P_\alpha(\alpha))=\operatorname{Res}(P'_\alpha,P_\alpha)$. Hence the second equation holds.

The rest of the proof is classical, as given in [B2]. We note that $\prod_{i>j}(\lambda_i-\lambda_j)$ is the determinant of the matrix (λ_i^q) with row number $i=1,\dots,n$ and column number $q=0,\dots,n-1$. When this matrix is multiplied from the left by its transpose the entry in position p,q is equal to the sum $\sum_{i=1}^n \lambda_i^{p+q}$, and consequently equal to $\operatorname{Tr}^N(\alpha^{p+q})$. Hence we have proved the Proposition. □

8.2. PROPOSITION. *Let N be a norm on a k-algebra A, and let α be an element in A. In the ring of power series in the variable t with coefficients in k we have the equation:*

$$(\tfrac{d}{dt}\log)N_{k[t]}(1-t\alpha):=\tfrac{d}{dt}N_{k[t]}(1-t\alpha)/N_{k[t]}(1-t\alpha)=-\sum_{j=0}^{\infty}\operatorname{Tr}^N(\alpha^{j+1})t^j.$$

Proof. Let n be the degree of the norm N. We have

$$N_{k[t]}(1-t\alpha)=\prod_{i=1}^{n}(1-\lambda_i t)$$

and consequently we have equations

$$(d\log/dt)N_{k[t]}(1-t\alpha)=\sum_{i=1}^{n}-\lambda_i/(1-t\lambda_i)=-\sum_{j=0}^{\infty}\sum_{i=1}^{n}\lambda_i^{j+1}t^j.$$

It follows from Corollary 7.2 that $\operatorname{Tr}^N(\alpha^l)=\sum_{i=1}^n\lambda_i^l$ for $l=1,2,\dots$. The formula of the Proposition follows. □

We note that, when k contains the rational numbers and N is a norm of degree n on a k-algebra A, we have that the characteristic polynomial $N_{k[t]}(t-\alpha)$ of an element α of A is determined by the elements $\mathrm{Tr}^N(\alpha^j)$ for $j=1,\dots,n$.

REFERENCES

[B1] BOURBAKI, N. *Algèbre.* Chapitre IV, Polynômes et fractions rationnelles. Masson, Paris, 1981.

[B2] —— *Algèbre.* Chapitre VII, Modules sur les anneaux principaux. Masson, Paris, 1981.

[L-S] LAKSOV, D. and R. M. SKJELNES. The Hilbert scheme parametrizing finite length subschemes of the line with support at the origin. To appear in *Compositio Math.*

[L] LANG, S. *Algebra.* 3rd edition. Addison-Wesley Publishing Company, 1993.

[M] MCCOY, N. H. *Rings and Ideals.* The Carus Mathematical Monographs 8. The Mathematical Association of America, 1948.

[R] ROBY, N. Lois polynômes et lois formelles en théorie des modules. *Ann. Sci. École Norm. Sup. 80* (1963), 213–248.

[S1] SKJELNES, R. M. On the representability of $Hilb^n k[x]_{(x)}$. To appear in *J. London Math. Soc.* (1999).

[S2] —— Symmetric tensors with applications to Hilbert schemes. To appear (1999).

(Reçu le 27 janvier 2000; version révisée reçue le 3 mai 2000)

Dan Laksov
L. Svensson

Department of Mathematics, KTH
KTH, S-100 44 Stockholm
Sweden
e-mail: laksov@math.kth.se
larss@math.kth.se

A. Thorup

Department of Mathematics, University of Copenhagen
Universitetsparken 5, DK-2100 København Ø,
Denmark
e-mail: thorup@math.ku.dk